1.(4 points) Evaluate $\iint_D y\,dA$ where D is the region in the first quadrant that lies above the hyperbola $xy = 1$ and the line $y = x$ and below the line $y = 2$.

Solution:

The region is obviously a type II region. We have

$$\iint_D y\,dA = \int_1^2 \int_{\frac{1}{y}}^{y} y\,dx\,dy = \frac{4}{3}$$

2.(3 points) Calculate the iterated integral by first reversing the order of the integration.

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3}\,dx\,dy$$

Solution:

Easy to see that the region is actually bounded $y = x^2, x = 1$ and the x-axis. So we have

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3}\,dx\,dy = \int_0^1 \int_0^{x^2} \frac{ye^{x^2}}{x^3}\,dy\,dx = \int_0^1 \frac{1}{2}xe^{x^2}\,dx = \frac{e-1}{4}$$

3.(3 points) Evaluate $\iint_D x\,dA$, where D is the region in the first quadrant the lies between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Solution:

Using polar coordinates, we have

$$\iint_D x\,dA = \int_0^{\frac{\pi}{2}} \int_1^2 r^2 \cos\theta\,dr\,d\theta = \int_0^{\frac{\pi}{2}} \cos\theta d\theta \int_1^2 r^2 dr = \frac{7}{3}$$