MATH 404 ANALYSIS - Spring 2008

HOMEWORK 6– Due Tuesday, April 10

1. Let Q be a closed rectangle in $\mathbb{R}^n$ and let $f : Q \to \mathbb{R}$ be continuous. Show that f is Riemann integrable.

2. Let $Q = [0,1] \times [0,1]$ and let $f : Q \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} 0 & x \neq y, \\ 1 & x = y. \end{cases}$$

Show that f is Riemann integrable over Q.

3. Let Q be a closed rectangle in $\mathbb{R}^n$ and let $f : [Q \to \mathbb{R}$. The graph of f is the subset

$$G(f) = \{(x,y) \in \mathbb{R}^{n+1} | y = f(x)\}$$

of $\mathbb{R}^{n+1}$. Show that if f is continuous, then G_f has measure zero in $\mathbb{R}^{n+1}$.

4. Let Q be a closed rectangle in $\mathbb{R}^n$ and let $f : Q \to \mathbb{R}$ be a bounded function. Show that if f vanishes except on a closed set B having measure zero, then $\int_Q f$ exists and is equal to 0.

5. Let Q be a closed rectangle in $\mathbb{R}^n$ and let $f : [Q \to \mathbb{R}$ be a bounded function. Assume that f is integrable over Q.

(a) Show that if $f(x) \geq 0$ for all $x \in Q$, then $\int_Q f \geq 0$.

(b) Show that if $f(x) > 0$ for all $x \in Q$, then $\int_Q f > 0$.