MATH 404 ANALYSIS - Spring 2008

HOMEWORK 4– Due Tuesday, March 4

1. Let $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}$ be defined by

$$f(x) = \sum_{i=1}^{n} |x_i| \quad \text{and} \quad g(x) = \max\{|x_i| \mid 1 \le i \le n\}.$$

Determine the set of points where f and g are differentiable.

2. Show that the function $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = |xy|$, is differentiable at $(0,0)$, but it is not of class C^1 on any open set containing $(0,0)$.

3. Show that if $A \subset \mathbb{R}^n$ and $f : A \to \mathbb{R}$, and if the partial derivatives $D_j f$ exist and are bounded on $B_\varepsilon(a) \subset A$, then f is continuous at a.

4. Let $g : \mathbb{R} \to \mathbb{R}$ be of class C^2. Show that

$$\lim_{h \to 0} \frac{g(a+h) - 2g(a) = g(a-h)}{h^2} = g''(a).$$

Hint: Consider the first step in the proof of the theorem about mixed derivatives in the case $f(x,y) = g(x+y)$.

5. Let $f : \mathbb{R}^3 \to \mathbb{R}$ and $g : \mathbb{R}^2 \to \mathbb{R}$ be differentiable. Let

$$F : \mathbb{R}^2 \to \mathbb{R}, \quad F(x,y) = f(x, y, g(x,y)).$$

(a) Find DF in terms of the partial derivatives of f and g.

(b) If $F(x,y) = 0$ for all (x,y), find $D_1 g$ and $D_2 g$ in terms of partial derivatives of f.