Problem 1. Let \( f : \mathbb{R}^n \to \mathbb{R}^n \) be defined by \( f(x) = |x|^2 \cdot x \). Show that \( f \) is of class \( C^\infty \), \( f \) is one-to-one and \( f(B_1(0)) = B_1(0) \). Show that the inverse function of \( f \) is not differentiable at 0.

Problem 2. Let \( g : \mathbb{R}^2 \to \mathbb{R}^2 \) be given by
\[
g(x, y) = (2ye^{2x}, xe^y),
\]
and let \( f : \mathbb{R}^2 \to \mathbb{R}^3 \) be given by
\[
f(x, y) = (3x - y^2, 2x + y, xy + y^3).
\]
(a) Show that there exist a neighborhood \( U \) of \((0, 1)\) and \( V \) of \((2, 0)\) such that \( g : U \to V \) is a bijection.
(b) Find \( D(f \circ g^{-1})(2, 0) \).

Problem 3. Let \( Q \) be a closed rectangle in \( \mathbb{R}^n \) and let \( f : Q \to \mathbb{R} \) be a bounded function. Prove:
(a) If \( f \) is Riemann integrable and \( f \) vanishes except on a set \( B \) having measure zero, then \( \int_Q f = 0 \).
(b) If \( f \) vanishes except on a closed set \( B \) having measure zero, then \( f \) is Riemann integrable and \( \int_Q f = 0 \).
(c) If \( f \) is Riemann integrable and \( f(x) > 0 \) for all \( x \in Q \), then \( \int_Q f > 0 \).

Problem 4. Show that if \( A \subset \mathbb{R}^n \) is compact and has measure 0 in \( \mathbb{R}^n \), then \( A \) has content 0 in \( \mathbb{R}^n \).

Problem 5. Let \( Q \) be a closed rectangle in \( \mathbb{R}^n \) and let \( f : Q \to \mathbb{R} \). The graph of \( f \) is defined by
\[
G = \{(x, y) \in \mathbb{R}^{n+1} | y = f(x)\}.
\]
Show that if \( f \) is continuous, then \( G \) has measure zero in \( \mathbb{R}^{n+1} \).