

MATH 404 ANALYSIS - Spring 2008

HOMEWORK 1– Due Tuesday, February 5

1. If $x = (x_1, x_2)$ and $y = (y_1, y_2) \in \mathbb{R}^2$, show that the function

$$\langle\langle x, y \rangle\rangle := x_1(ay_1 + by_2) + x_2(by_1 + cy_2)$$

is an inner product on \mathbb{R}^2 if and only if $b^2 - ac < 0$ and $a > 0$.

2. Determine A^0 , \bar{A} and ∂A for the subset A of \mathbb{R}^2 given by

$$\begin{array}{ll} \{x, y \mid x = 0\} & \{(x, y) \mid x \text{ is rational and } y > 0\} \\ \{(x, y) \mid 0 \leq y < 1\} & \{(x, y) \mid x \text{ and } y \text{ are rational}\}. \end{array}$$

3. Let $V = (-1, 0) \cup (0, 1]$. Show that the sets $(-1, 0)$ and $(0, 1]$ are open and closed in V . Show also that $A = (-1/2, 1/2) \cap V$ is open but not closed in V .

4. Let $f : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ be defined by $f(\alpha, x) = \alpha x$. Show that f is continuous.

5. Define $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ by

$$f(x_1, x_2) = \begin{cases} \frac{x_1 x_2^2}{x_1^2 + x_2^6} & x \neq 0 \\ 0 & x = 0. \end{cases}$$

Show that f is not continuous.