MAT 250 Practice Exam.

1. Assume that eigenvectors of a 2×2 matrix $\mathbf{A}$ are $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$. Assume that the first of these vectors corresponds to the eigenvalue -1 and the second corresponds to the eigenvalue 0.

(a) Determine the general solution of the system $\mathbf{y}' = \mathbf{A}\ \mathbf{y}$.

(b) Describe the behavior of the solution as $t \longrightarrow -\infty$.

(c) Find a solution of the system that, in addition, satisfies the initial condition $\mathbf{y}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

2. Find the general solution of the following system:

$$\mathbf{y}' = \begin{pmatrix} 2 & -9 \\ 1 & 2 \end{pmatrix} \mathbf{y}.$$

Describe the behavior of the solution as $t \longrightarrow \infty$.

3. Find the general solution of the following system:

$$\mathbf{y}' = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix} \mathbf{y}.$$

Describe the behavior of the solution as $t \longrightarrow \infty$.

4. Find the general solution of the following system:

$$\mathbf{y}' = \begin{pmatrix} 6 & 2 \\ 2 & 6 \end{pmatrix} \mathbf{y} + \begin{pmatrix} e^{8t} \\ -2e^{-t} \end{pmatrix}.$$

5. Give an example of a critical point y_0 of a linear system such that y_0 is unstable but, nevertheless, $\lim_{t \longrightarrow \infty} y(t) = y_0$ for some solutions $y(t)$.

6. (a) Solve the differential equation

$$(3 - t^2)y'' - 3ty' - y = 0$$

by means of a power series about $t_0 = 0$. Find the first six terms in each of two linearly independent solutions.

(b) Estimate the radius of convergence of the series.

7. Find the general solution of the system

$$\mathbf{y}' = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix} \mathbf{y} + \begin{pmatrix} t \\ -3 \end{pmatrix}.$$

8. Solve the initial value problem

$$2y'' - e^y = 0; y(0) = 0; y'(0) = 1.$$

Proceed as follows: 1) Multiply both sides of the equation by y'; 2) Observe that $y''y' = \frac{1}{2}((y')^2)'$ and $e^y y' = (e^y)'$ and integrate the resulting equation; 3) Use the initial conditions to determine the constant; 4) Find the general solution of the resulting first-order equation using separation of variables; 5) Use the initial conditions to determine the solution.