MAT 250 ODE Sample Test 1

Multiple choice problems.

1. (15 points.) Assume that the initial value problem \( y' = 1 - y^2; y(0) = 0 \) is being solved for \( t \) in the interval \([0, 3]\) using Euler’s method with \( h = 1 \). That is, one takes \( t_0 = 0, t_1 = 1; t_2 = 2, t_3 = 3 \) and finds the approximate values \( y_0, y_1, y_2, y_3 \) of \( y \) at \( t_0, t_1, t_2, t_3 \). What will be the value of \( y_3 \)?
   A. 0; B. 1; C. \( -\frac{2}{3} \); D. 2; E. 2.

2. (15 points) Let \( y(t) \) denote the solution of the initial value problem \( y' - y = t; y(0) = 0, t > 0 \). Then \( y(1) \) is equal to:
   A. \( e \); B. \( e - 2 \); C. \( \frac{2}{\pi} \); D. 0; E. 1.

3. (15 points) Let \( y(t) \) be the solution of the initial value problem \( y' = \frac{3x^2 - 1}{3x^3 + 1} \);
   \( y(1) = 1 \). Then \( y(-1) \) is equal to
   A. \(-1\); B. \( 2 \); C. \( 2^\frac{1}{4} - 1 \); D. 1; E. 0.

4. (15 points) \( y_1(x) = \exp(x) \) is a solution of the differential equation \( (x - 1)y'' - xy' + y = 0, x < 1 \). Use this information to find \( y(1) \), where \( y(t) \) is the solution of the initial value problem \( (x - 1)y'' - xy' + y = 0, y(0) = 0, y'(0) = 1, x < 0 \). The value of \( y(1) \) is equal to
   A. \(-1\); B. \( e \); C. \( \frac{1}{e} \); D. 1; E. 1 - \( e \).

Open ended problems.

5. (15 points) Using a direction field sketch the graph of the solution of the initial value problem \( y' = t - \sin y; y(0) = 0 \). How does \( y(t) \) behave as \( t \) becomes large?

6. Find the general solutions of the following differential equations:
   (a) (7 points) \( y'' + 3y' + 2y = 0 \);
   (b) (8 points) \( 5y'' + 13y' + 9y = 0 \);
   (c) (10 points) \( y'' - 6y' + 9y = 0 \).