1. (6 points) Let T be the period, R the amplitude, and θ the phase of the function

$$u(t) = \cos(3t) + \sin(3t).$$

Then

(a) $T = 3/(2\pi)$, $R = 1$, $\theta = 0$

(b) $T = 6\pi$, $R = \sqrt{2}$, $\theta = \pi$

(c) $T = 2\pi/3$, $R = 1$, $\theta = \pi$

(d) $T = 2\pi/3$, $R = \sqrt{2}$, $\theta = \pi/4$

2. (6 points) Let $y(t)$ be any solution of the differential equation

$$y'' + 2y' + y = 1.$$

Then

(a) $\lim_{t\to+\infty} y(t) = 0$

(b) $\lim_{t\to+\infty} y(t) = +\infty$

(c) $\lim_{t\to+\infty} y(t) = 1$

(d) $\lim_{t\to+\infty} y(t)$ cannot be determined

3. (6 points) A suitable form for a particular solution $y(t)$ to the differential equation

$$y'' + 2y' + 2y = t\cos(2t)$$

is:

(a) $Ae^{-t}\cos(t) + Be^{-t}\sin(t)$

(b) $At\sin(2t) + Bt\cos(2t)$

(c) $(A_1t + A_2)\sin(2t) + (B_1t + B_2)\cos(2t)$

(d) $(A_1t^3 + A_2t^2)\sin(2t) + (B_1t^3 + B_2t^2)\cos(2t)$

4. (6 points) A spring is stretched $L = 0.1$ m by an object of mass $m = 1$ kg. The system is set in motion at time $t = 0$ by an external force of $F(t) = \cos(kt)$ Newtons. Assume that there is no damping force and that $g = 10$ m/sec^2. For which value of k does resonance occur:

(a) $k = \sqrt{10}$

(b) $k = 10$

(c) $k = 100$

(d) resonance never occurs

5. (6 points) Find the inverse Laplace transform of

$$F(s) = \frac{s}{s^2 + 2s + 2}$$

(a) $e^{-t}(\cos(t) + \sin(t))$

(b) $e^{-t}(\cos(t) - \sin(t))$

(c) $e^{-t}\cos(t)$

(d) $e^{-t}(\cos(t) - 2\sin(t))$

6. (12 points) Find the Laplace transform of

$$f(t) = \begin{cases} 3 & 0 \leq t < 5 \\ t & 5 \leq t \end{cases}$$

7. (13 points) Find the general solution to the differential equation

$$x^2y'' + 3xy' + y = 0 \qquad x > 0$$

given that $y = x^{-1}$ is a solution.

8. (18 pts) Find the general solution of (a), (b)

(a) $y'' + 4y = \sin(2t)$

(b) $y'' - 6y' + 9y = 0$

(c) Give the *form* of a particular solution of $y'' - 6y' + 9y = e^{3t}$. Do not solve for it!

9. (12 points) Consider the non-homogeneous linear differential equation

$$y'' + p(t)y' + q(t)y = g(t) \tag{1}$$

where p, q, g are continuous functions.

(a) If $y_1(t)$ and $y_2(t)$ are solutions of (1) show that $w(t) = y_1(t) - y_2(t)$ is a solution of the corresponding homogeneous equation $y'' + p(t)y' + q(t)y = 0$.

(b) Suppose that

$$y_1(t) = e^t, \quad y_2(t) = e^t + \cos t, \quad y_3(t) = e^t + \sin t$$

are three solutions of the non-homogeneous equation (1). Find the general solution of (1).

10. (15 points) Suppose a mass-spring system has mass $m = 1$ kg, damping constant $\gamma = 4$ kg/sec, and spring constant $k = 3$ kg/sec^2. Assume that at time zero, the mass is released with the spring *compressed* 2 cm from the mass's equilibrium position, and an instantaneous velocity of 5 cm/sec in the direction of spring *decompression* is imparted upon the mass.

    (a) Write an initial value problem describing this situation.

    (b) Solve your problem in (a) for the equation of motion.

    (c) How many times does the mass cross the equilibruim position?