NAME :_____

ID No :_____

MATH 250
Fall 2005
Exam 1

SECTION :_____

This exam contains 10 questions on 9 pages (including this title page). This exam is worth a total of 100 points. The exam is broken into two parts. There are six multiple choice question, each worth 5 points, and 4 partial credit problems. To receive full credit for a partial credit problem all work must be shown. When in doubt, fill in the details.

No notes, books or calculators may be used during this exam.

Please Box Your Final Answers (when possible).

1:_____
2:_____
3:_____
4:_____
5:_____
6:_____
7:_____
8:_____
9:_____
10:_____
Total:_____

Multiple Choice Section

1. (5 points) Which of the following ordinary differential equations is linear of the second order?

   (a) $y'' + 4yy' = 0$

   (b) $t^2y'' + ty' - y = \ln t$

   (c) $(y')^2 + 2y = 0$

   (d) $t^2 + y' = e^{-t}$

2. (5 points) Which of the following will be an integrating factor for the differential equation

$$ty' - 2y = 2\cos 2t?$$

   (a) $\frac{1}{t^2}$

   (b) e^{-2t}

   (c) $-t^2$

   (d) $-2t$

3. (5 points) Find the maximal interval in which the existence and uniqueness of the solution to the following initial value problem is guaranteed:

$$(\sin t)y' + \frac{1}{t-1}y = \frac{t}{t+1}, \qquad y(2) = 3.$$

(a) $(0, \pi)$

(b) $(0, 2)$

(c) $(-1, \pi)$

(d) $(1, \pi)$

4. (5 points) Find a second order linear differential equation with constant coefficients to which the following functions are solutions:

$$y_1(t) = e^{-2t}, \quad y_2(t) = e^t.$$

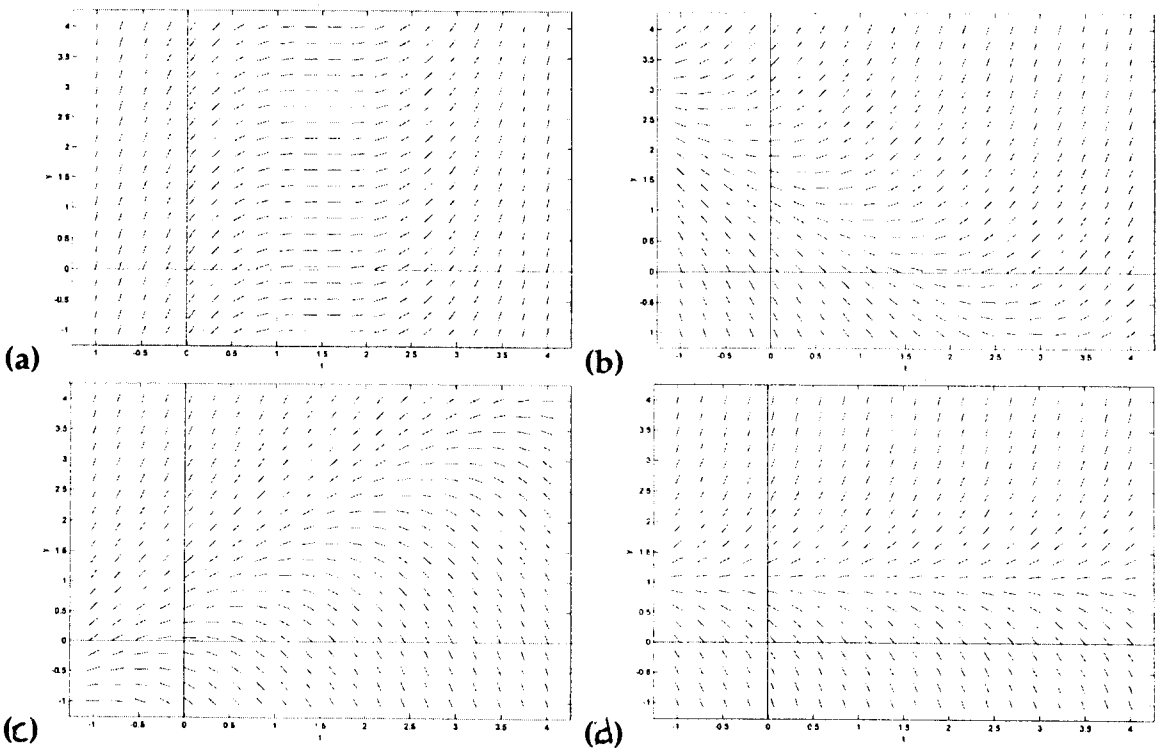
(a) $y'' - y' - 2y = 0$

(b) $y'' + 2y' - 3y = 0$

(c) $y'' + y' - 2y = 0$

(d) $y'' + y' + 2y = 0$

5. (5 points) Which of the following is the direction field for the differential equation $y' = y - t$?



6. (5 points) Consider the following two pairs of solutions:

$$y_1(t) = e^t \quad \text{and} \quad y_2(t) = e^{t+1}$$
$$z_1(t) = \sin t \quad \text{and} \quad z_2(t) = \sin(2t)$$

Which of the following is true?

(a) Both of these pairs form fundamental sets of solutions.

(b) Only the second pair forms a fundamental set of solutions.

(c) Only the first pair forms a fundamental set of solutions.

(d) Neither pairs form fundamental sets of solutions.

Partial Credit Section

7. (a) (10 points) Solve the initial value problem

$$y' = \frac{x^3 + 1}{yx^2 - x^2}, \qquad y(1) = 0.$$

Write your answer y as an explicit function of x.

(b) (5 points) Find the maximal possible interval (a, b) on which the above solution is valid. If it is difficult to solve for a or b without a calculator, you may describe it as a zero of a certain function.

8. (20 points) A tank initially contains 60 gal of pure water. Sweet water containing 1 lb of sugar per gallon enters the tank at 2 gal/min, and the (perfectly mixed) solution leaves the tank at the same rate.

   (a) (13 points) Set up and solve an initial value problem for the amount of sugar Q at any time t.

   (b) (7 points) Find an instant of time T when the concentration of the sugar is $1/2$ lb per gal.

9. (20 points) Consider the following autonomous equation:

$$y' = y^2 - 4y.$$

(a) (3 points) Sketch the graph of $f(y) = y^2 - 4y$.

(b) (6 points) Find the critical points and classify equilibrium solutions for this differential equation.

(c) (3 points) Find inflection points.

Question 9 continued.

(d) (4 points) Sketch the graphs of several solutions, making sure you have at least one graph representing each type.

(e) (4 points) Find the limit as $t \to \infty$ of the solution satisfying the initial condition $y(1) = 3$, without solving the equation.

10. (a) (10 points) Find the general solution to the second order differential equation

$$y'' + y' - 2y = 0.$$

(b) (5 points) Find the value β such that the solution in (a) satisfying the initial conditions

$$y(0) = 3, \qquad y'(0) = \beta$$

remains finite as the independent variable $t \to \infty$.