

5th Geometry and Physics Conference

Dakar, 14-19 May, 2007

Université Cheikh Anta Diop

Abstracts of mini-courses

ABSTRACTS

Anton Alekseev (University of Geneva, Switzerland)

CAMPBELL-HAUSDORFF SERIES AND THE KASHIWARA-VERGNE CONJECTURE

ABSTRACT: TBA

Augustin Banyaga (Penn State University, USA)

INTRODUCTION TO THE GEOMETRY OF HAMILTONIAN Diffeomorphisms

We will start with a general exposition of the basics on symplectic diffeomorphisms, hamiltonian diffeomorphism and the flux homomorphism. Then, we present some fundamental theorems (the simplicity of hamiltonian diffeomorphisms (Banyaga), the C^0 -rigidity of hamiltonian diffeomorphisms (Ellashgerb-Gromov), the Arnold conjecture and Floer homology, the solution by Ono of the flux conjecture). Finally, we will introduce the Hofer metric on the group of hamiltonian diffeomorphisms. At the end of the mini-course, we will present a list of open problems.

Jean-Paul Dufour (Université Montpellier 2, France)

POISSON STRUCTURES AND PROBLEMS CONCERNING THEIR SINGULARITIES

The first two lectures of this course will be devoted to basic concepts and results in Poisson geometry. In the last lecture, we will present open problems on singularities of Poisson structures. Here is a more detailed outline of the lecture topics:

First Lecture:

- Definitions of Poisson structures and Poisson tensors
- Examples of symplectic structures and Lie-Poisson structures
- Hamiltonian vector fields and Poisson morphisms
- Symplectic foliation

Second Lecture:

- Darboux-Weinstein coordinates
- Transversal Poisson structure to a symplectic leaf
- Schouten bracket
- Curl of a Poisson tensor
- Poisson cohomology. Using Poisson cohomology for the study of Poisson singularities

Third Lecture:

- Known results concerning the linearization of Poisson structures near a singularity
 - Quadratic structures of Poisson and results of quadratization
 - Problems on the quadratization of the structures transversal to the co-adjoint orbits
 - Stable singularities and the theorem of Crainic-Fernandes; open problems concerning the local stability of decomposable Poisson structures
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Tudor Ratiu (École Polytechnique Fédérale de Lausanne, Suisse)

SLICE THEOREM IN INFINITE DIMENSIONS AND HYDRODYNAMICS WITH
CIRCULAR SYMMETRY

The general definition and properties of manifolds, maps and groups of diffeomorphisms of class H^s will be presented. Then the key geometric results about the ideal incompressible homogeneous Euler equations will be given. A slice theorem for a finite dimensional Lie group on a manifold of maps will be stated. Then it will be applied to fluid motion with circular symmetry.

Reyer Sjamaar (Cornell University, USA)

REAL SYMPLECTIC GEOMETRY

A *real* symplectic manifold is a symplectic manifold equipped with an antisymplectic involution. Its *real locus* is the fixed-point set of the involution, which is a Lagrangian submanifold. This notion derives its name from the special case of a complex projective variety defined over the real numbers. Slightly less obviously, many quaternionic objects, such as quaternionic flag manifolds, are also real loci of symplectic manifolds.

A time-tested way to gain understanding of a real variety is to study the relationship to its complexification. This idea has turned out to be fruitful in the symplectic category also. If G is a Lie group, a *real Hamiltonian G -manifold* is a real symplectic manifold equipped with a Hamiltonian G -action that is $\mathbb{Z}/2$ -equivariant with respect to an involution on G .

The study of real Hamiltonian G -manifolds in the case where G is a torus was initiated by Duistermaat, who extended to the real case several fundamental results from symplectic geometry, such as the cohomological perfection of the moment map and the abelian convexity theorem due to Atiyah and Guillemin-Sternberg.

The general notion, which involves symmetric spaces and restricted root systems in an interesting way, was introduced by me and my former student O'Shea for the purpose of obtaining real forms of the nonabelian convexity theorem of Kirwan and the eigenvalue inequalities of Klyachko. The quotient of a real Hamiltonian G -manifold is a real symplectic manifold (when nonsingular). Important examples of real Hamiltonian G -manifolds are polygon spaces, the various types of real and quaternionic flag manifolds, and (in infinite dimensions) the space of connections on a principal bundle on a *nonorientable* Riemann surface.

In my talks, I will discuss convexity properties of real Hamiltonian G -manifolds, as well as joint work with Tara Holm concerning their cohomological properties.

Pol Vanhaecke (Université de Poitiers, France)

ALGEBRAIC INTEGRABILITY

Most integrable systems of interest admit a natural complexification in which the Liouville tori become complex algebraic tori (Abelian varieties). Far from being artificial, the introduction of complex time and complex coordinates reveals a rich underlying algebraic and geometric structure by means of which constructions such as linearization, action-angle variables, explicit solutions, hidden symmetries, not only become feasible for particular examples, but even become systematic for several general classes of examples. Starting from the general notion of Liouville integrability on a Poisson manifold (real or holomorphic) and after introducing the rudiments on Abelian varieties I will introduce algebraic integrable systems, illustrate their basic properties on some first examples, and provide some tools to explore their geometry. Prerequisites are basic notions on manifold theory (real, complex, algebraic) and familiarity with Poisson structures on (such) manifolds.