MATH 220 FINAL EXAMINATION DECEMBER 16, 2004

Name______________________________ ID #__________________ Section #______

There are 25 multiple choice questions. Four possible answers are given for each problem, only one of which is correct. When you solve a problem, note the letter next to the answer that you wish to give and blacken the corresponding space on the answer sheet. **Mark only one choice; darken the circle completely** (you should not be able to see the letter after you have darkened the circle).

THE USE OF CALCULATORS DURING THE EXAMINATION IS FORBIDDEN.

PLEASE SHOW YOUR PSU ID CARD TO YOUR INSTRUCTOR WHEN YOU FINISH.
GOOD LUCK.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 25 PROBLEMS ON 14 PAGES (INCLUDING THIS ONE).

1. Consider the matrices $A = \begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 & 5 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. Which of the following statements is true?

   a) None of the matrices is in reduced echelon form

   b) Both A and B are in reduced echelon form

   c) Only A is in reduced echelon form

   d) Only B is in reduced echelon form

2. What is the general solution of the linear system $\begin{cases} 2x_1 + x_2 + x_3 = 10 \\ x_2 - x_3 = 4 \\ x_1 - 2x_2 + 3x_3 = -5 \end{cases}$?

   a) $\begin{cases} x_1 = 3 - 2x_3 \\ x_2 = 4 + x_3 \\ x_3 \text{ is free} \end{cases}$

   b) $\begin{cases} x_1 = 1 - 2x_2 \\ x_3 = 4 - x_2 \\ x_2 \text{ is free} \end{cases}$

   c) $\begin{cases} x_1 = 3 - x_3 \\ x_2 = 4 + x_3 \\ x_3 \text{ is free} \end{cases}$

   d) None of the above

3. If T is a linear transformation whose standard matrix is given by $A = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ 5 & 1 \end{bmatrix}$, then which of the following statements is true?

a) T is one-to-one, but not onto.

b) T is not one-to-one, but it is onto.

c) T is both one-to-one and onto.

d) T is neither one-to-one nor onto.

4. Let $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & -2 & -6 \\ -1 & 3 & 10 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Then $A\mathbf{x} = \mathbf{b}$ is **consistent** if

a) $2b_1 + 5b_2 + 2b_3 = 0$

b) $2b_1 - b_2 + 5b_3 = 0$

c) $b_1 - 3b_2 + b_3 = 0$

d) $b_1 + b_2 + b_3 = 0$

5. Find the parametric vector form of the solution set of the system $\begin{cases} x_1 + x_2 - 2x_3 &= 5 \\ 2x_1 + 3x_2 + 4x_3 &= 2 \end{cases}$

a) $\mathbf{x} = x_3 \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

b) $\mathbf{x} = \begin{bmatrix} 13 \\ -8 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 10 \\ -8 \\ 1 \end{bmatrix}$

c) $\mathbf{x} = x_2 \begin{bmatrix} 13 \\ -8 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

d) None of the above

6. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$. Which of the following statements is true?

a) Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is the origin.

b) Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a line through the origin.

c) Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a plane through the origin.

d) Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is all of $\mathbb{R}^3$.

7. Consider the two planes given by $2x_1 + 3x_2 - 5x_3 = 7$ and $x_1 - x_2 + 5x_3 = 1$. Which of the following statements is true?

   a) Their intersection is empty.

   b) Their intersection is the point $(2, 1, 0)$.

   c) Their intersection is a line through the point $(-2, 3, 1)$ with direction $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

   d) None of the above statements describes the intersection of these planes.

8. Let A be the standard matrix of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ that is *onto*. Which of the following statements is true?

   a) A is not invertible.

   b) $A\mathbf{x} = 0$ has a only the trivial solution.

   c) The columns of A are linearly dependent.

   d) T is *not* one-to-one.

9. Suppose T is the linear transformation that first rotates points through $\frac{\pi}{4}$ radians counterclockwise and then projects points onto the x-axis. The standard matrix of T is

a) $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

d) None of the above

10. Which of the following sets is a subspace of $\mathbb{R}^3$?

a) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid x_1 + x_2 + x_3 = 7 \right\}$

b) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid x_2 + x_3 = 0 \right\}$

c) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid \sin(x_2) - x_3 = 0 \right\}$

d) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid x_1^2 + x_2^2 = 0 \right\}$

11. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 4 & 2 \\ -1 & -2 & 5 & 3 \end{bmatrix}$. What is the dimension of the null-space of A?

a) 1

b) 2

c) 3

d) 4

12. Let $A = \begin{bmatrix} 2 & -3 & 0 & 2 \\ 1 & 0 & 3 & 1 \\ 0 & 2 & 4 & 0 \end{bmatrix}$. Find a basis for the column space of A.

a) $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$,

b) $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} \right\}$

d) None of the above

13. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ -1 & 5 & 11 \end{bmatrix}$. Which of the following vectors is in the null-space of A?

a) $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

b) $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

14. What is the characteristic polynomial of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$?

a) $(1-\lambda)(1+\lambda)(\lambda-3)$

b) $(1-\lambda)^2(\lambda+1)$

c) $(1-\lambda)(\lambda+2)$

d) None of the above

15. The determinant of $A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is

a) 24

b) -24

c) 0

d) None the above.

16. Let A and B be 3×3 matrices such that $\det(A) = 4$ and $\det(B) = 3$. Find $\det(2A^{-1}B^2)$.

a) $\frac{9}{2}$

b) 72

c) -18

d) 18

17. Find the eigenvalues of $A = \begin{bmatrix} 1 & 5 & 3 \\ 0 & 3 & 0 \\ 0 & 2 & 2 \end{bmatrix}$.

a) $\lambda = 1, 2, 3$

b) $\lambda = 0, 1, 2$

c) $\lambda = 1, 3, 5$

d) $\lambda = 2, 3, 5$

18. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation whose standard matrix is $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$. Find a basis $\mathcal{B}$ such that the $\mathcal{B}$-matrix of T is diagonal.

a) $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

b) $\mathcal{B} = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

c) $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$

d) $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

19. Which of the following sets is an *orthonormal* set?

a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1/\sqrt{3} \\ 2/\sqrt{3} \end{bmatrix} \begin{bmatrix} 2/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} -2/\sqrt{6} \\ 1/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

20. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$. Then A^3 equals

a) $\begin{bmatrix} -29 & 56 \\ -28 & 55 \end{bmatrix}$

b) $\begin{bmatrix} -24 & 56 \\ -25 & 55 \end{bmatrix}$

c) $\begin{bmatrix} -29 & 56 \\ -28 & 57 \end{bmatrix}$

d) None of the above

21. Let $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$. Find the coordinate vector of $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ relative to the *orthogonal* basis $\mathcal{B}$ for $\mathbb{R}^3$

a) $\begin{bmatrix} 1/6 \\ 5/2 \\ 2/3 \end{bmatrix}$

b) $\begin{bmatrix} 1/6 \\ 5/2 \\ 2 \end{bmatrix}$

c) $\begin{bmatrix} 1/6 \\ 5/2 \\ 2/3 \end{bmatrix}$

d) None of the above

22. What is the distance between $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$ and the **line** L through the origin that is spanned by $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$?

a) $\sqrt{14}$

b) $\sqrt{26}$

c) $\sqrt{6}$

d) None of the above

23. A diagonalization of $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ is

a) $\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$

c) $\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

d) None of the above

24. Which of the following statements is *false*?

a) An $n \times n$ matrix with n distinct real eigenvalues is diagonalizable.

b) If A is invertible then A is diagonalizable.

c) Similar matrices have the same determinant.

d) Similar matrices have the same eigenvalues.

25. Use the Gram-Schmidt process to find an *orthonormal* basis for $H = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix} \right\}$.

a) $\left\{ \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{14} \\ 3/\sqrt{14} \\ -2/\sqrt{14} \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} \right\}$