MATH 220 FINAL EXAMINATION December 19, 2006

Name______________________________ ID #__________________ Section #______

There are ??multiple choice questions. Each problem is worth 6 points. Four possible answers are given for each problem, only one of which is correct. When you solve a problem, note the letter next to the answer that you wish to give and blacken the corresponding space on the answer sheet. **Mark only one choice; darken the circle completely** (you should not be able to see the letter after you have darkened the circle).

THE USE OF CALCULATORS DURING THE EXAMINATION IS FORBIDDEN.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 25 PROBLEMS ON 14 PAGES (INCLUDING THIS ONE).

1. Find all value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & h & -2 \\ -3 & 7 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & h & -2 \\ 0 & 7+3h & 0 \end{bmatrix}$$

a) $-7/3$

b) $7/3$

c) all $h \in \mathbb{R}$

d) no $h \in \mathbb{R}$

2. Which of the following matrices is in the reduced echelon form?

a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 5 & 0 \end{bmatrix}$.

b) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

c) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

3. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -2 \\ -2 \\ 7 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ h \end{bmatrix}$. For what value(s) of h is $\mathbf{b}$ in the plane spanned by $\mathbf{a}_1$ and $\mathbf{a}_2$?

a) $h = -10$

b) $h = -5$

c) $h = -2$

d) $h = -1$

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 3 & -2 & 1 \\ -4 & 7 & h \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 \\ 0 & 4 & -8 \\ 0 & -1 & h+12 \end{bmatrix}$$

$\text{Row}_2 \to \text{Row}_2 - 3\,\text{Row}_1$

$\text{Row}_3 \to \text{Row}_3 + 4\,\text{Row}_1$

$$A \sim \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -2 \\ 0 & -1 & h+12 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & h+10 \end{bmatrix}$$

Consistent if $h+10=0$, that is, $h=-10$

4. Find the general solution to the linear system corresponding to the augmented matrix

$$\begin{bmatrix} 1 & 2 & 2 & 1 & -2 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$

a) $x_1 = 2x_3$, $x_2 = x_4 + 1$, and x_3, x_4 are free.

b) $x_1 = -2x_3$, $x_2 = -x_4 + 1$, and x_3, x_4 are free.

c) $x_1 = 2x_2 + x_4$, $x_3 = -1$, and x_2, x_4 are free.

d) $x_1 = -2x_2 - x_4$, $x_3 = -1$, and x_2, x_4 are free.

5. Consider the linear system:

$$\begin{cases} 3x_1 - 6x_2 + x_3 = -1, \\ 2x_1 - 4x_2 + x_3 = 0. \end{cases}$$

Which of the following is the parametric vector form of its solutions?

a) $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \; t \in \mathbb{R}.$

b) $\begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \; t \in \mathbb{R}.$

c) $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \; t \in \mathbb{R}.$

d) $\begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \; t \in \mathbb{R}.$

$$\begin{bmatrix} 3 & -6 & 1 & -1 \\ 2 & -4 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & -6 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$R_2 \to 2R_1 - 3R_2$

$$X = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + X_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Take $X_2 = t$

6. If the set $\left\{ \underbrace{\begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix}}_{a_1}, \underbrace{\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}}_{a_2}, \mathbf{v} \right\}$ is linearly independent, what is a possible vector $\mathbf{v}$?

X ~~a)~~ $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

X c) $\begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix} = a_1 + 3a_2$

X d) $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = a_1 + a_2$

7. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation and $T(\mathbf{e}_1 + \mathbf{e}_2) = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, $T(\mathbf{e}_1 - \mathbf{e}_2) = \begin{bmatrix} -4 \\ 3 \\ -2 \end{bmatrix}$, where $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. What is $T\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right)$?

a) $\begin{bmatrix} -5 \\ 3 \\ -7 \end{bmatrix}$

b) $\begin{bmatrix} 7 \\ -3 \\ 5 \end{bmatrix}$

c) $\begin{bmatrix} 5 \\ -3 \\ 7 \end{bmatrix}$

d) $\begin{bmatrix} -7 \\ 3 \\ -5 \end{bmatrix}$

$T\begin{bmatrix} -1 \\ 2 \end{bmatrix} = -T(e_1) + 2T(e_2)$

$T(e_1 + e_2) = T(e_1) + T(e_2) = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

$T(e_1 - e_2) = T(e_2) - T(e_2) = \begin{bmatrix} -4 \\ 3 \\ -2 \end{bmatrix}$

Therefore $2T(e_1) = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} + \begin{bmatrix} -4 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ 2 \end{bmatrix}$

$T(e_1) = \frac{1}{2}\begin{bmatrix} -2 \\ 6 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$

$2T(e_2) = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} -4 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix}$

$-T(e_1) + 2T(e_2) = \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix} - \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 5 \end{bmatrix}$

8. Let A be an $m \times n$ matrix, and $T : \mathbb{R}^n \to \mathbb{R}^m$ the linear transformation whose standard matrix is A. Which of the following statements is not always true?

a) If $m < n$, then T cannot be one-to-one.

b) If $m > n$, then T cannot map $\mathbb{R}^n$ onto $\mathbb{R}^m$.

c) If $m = n = 3$, and T maps $\mathbb{R}^3$ onto $\mathbb{R}^3$, then T is always one-to-one.

d) If $m \neq n$ and T is one-to-one, then A has m pivot positions.

9. Suppose A, B and C are 4×4 matrices. Which of the following is (are) always true?

✓ (1) $(A+B)^T = A^T + B^T$
✗ (2) $(A+B)^{-1} = A^{-1} + B^{-1}$
(3) $(AB)^T = A^T B^T$
✓ (4) $(AB)^{-1} = B^{-1} A^{-1}$

$(AB)^T = B^T A^T$

a) (1), (2) and (3).

b) (1), (2) and (4).

c) (1) and (4).

d) all of the above.

10. Let $A = \begin{bmatrix} 2 & 4 & -2 \\ 1 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$. Then the first row of A^{-1} is

a) $[1/8 \quad -1/2 \quad 5/4]$.

b) $[1/8 \quad 1/8 \quad -1/8]$.

c) $[1/8 \quad 1/2 \quad 3/4]$.

d) The matrix is not invertible.

Let

$$A^{-1} = \begin{bmatrix} a & b & c \\ * & * & * \\ * & * & * \end{bmatrix}$$

We must find a, b and c

$$A^{-1}A = \begin{bmatrix} a & b & c \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} 2 & 4 & -2 \\ 1 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2a+b+c & 4a+b & -2a+2b+c \\ * & * & * \\ * & * & * \end{bmatrix}$$

But $A^{-1}A = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence $\begin{cases} 2a+b+c = 1 \\ 4a+b = 0 \\ -2a+2b+c = 0 \end{cases}$

After solving the system one gets

$$\boxed{a = 1/8, \quad b = -1/2 \quad c = \frac{5}{4}}$$

11. Which of the following statements is (are) true?

(1) If A^2 is invertible, then the columns of A^T are linearly independent.
(2) If A is invertible, then the columns of A are linearly independent.
(3) A triangular matrix is always invertible.

a) (1) only.

b) (1) and (3) only.

c) (1), (2) and (3).

d) (1) and (2) only.

12. What is the coordinate vector of $\mathbf{x} = \begin{bmatrix} 3 \\ 10 \\ -8 \end{bmatrix}$ relative to the basis $\left\{ \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix} \right\}$?

a) $\begin{bmatrix} -1/2 \\ 2 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} -1/2 \\ 2 \end{bmatrix}$

c) $\begin{bmatrix} 3 \\ 10 \end{bmatrix}$

d) None of the above.

Solve $X = x_1 \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}$

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 2 \end{bmatrix}$

13. Given that $A = \begin{bmatrix} 1 & 2 & -4 & 3 & 3 \\ 5 & 10 & -9 & -7 & 8 \\ 4 & 8 & -9 & -7 & 8 \\ -2 & -4 & 5 & 0 & -6 \end{bmatrix}$ is row equivalent to $\begin{bmatrix} 1 & 2 & -4 & 3 & 3 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & -5 & -4 \\ 0 & 0 & 0 & 0 & -7 \end{bmatrix}$, what is a basis for Col A?

a) $\left\{ \begin{bmatrix} 1 \\ 5 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ -9 \\ -9 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ 8 \\ -6 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ -4 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -5 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 2 \\ -4 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 10 \\ -9 \\ -7 \\ 8 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ -9 \\ -7 \\ 8 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -5 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -4 \\ -7 \end{bmatrix} \right\}$