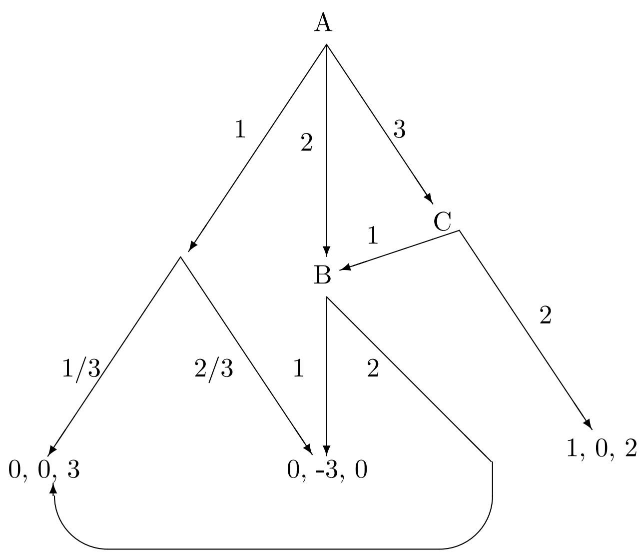
Find:
normal form,
the equilibria in pure strategies,
the Pareto optimal pure payoffs,
the Nash bargaining solution,
the Shapley values,
an imputation in the core.



See the next page for solutions.

Solution. There are 12 strategy profiles:

strategy A B C	payoff A B C
1 1 1	0 - 2 1
1 1 2	0 - 2 1
1 2 1	0 - 2 1
1 2 2	0 - 2 1
2 1 1	0 - 3 0
2 1 2	0 - 3 0
2 2 1	0 0 3
2 2 2	0 0 3
3 1 1	0 - 3 0
3 1 2	1 0 2
3 2 1	0 0 3
3 2 2	1 0 2

There are 5 equilibria in pure strategies: (1,1,1), (1,2,1), (2,2,1), (3,1,2), and (3,2,1).

There are 2 Pareto optimal pure payoffs: (0, 0, 3) and (1, 0, 2).

The characteristic function v is :
v(empty) = 0, v(A,B,C) = 3,
v(A) = 0, v(B) = -2, v(C) = 0,
v(A,B) = 0, v(A,C) = 3, v(B,C) = -1.

The initial point for the Nash bargaining: (v(A),v(B), v(C)) = (0, -2, 0).

$$a(b+2)c \to \max, a \geq 0, b \geq -2, c \geq 0,$$

where (a, b, c) is a mixture of (0, 0, 3) and (1, 0, 2) so $b = 0$. Since $a + c = 3$ and we want to maximize ac, the optimal solution on the line ist $a = c = 3/2$. It is outside the interval of Pareto optimal solutions. The closest point in the interval is the arbitration triple (1, 0, 2).

	A	B	C
ABC	0	0	3
ACB	0	0	3
BAC	2	-2	3
BCA	4	-2	1
CAB	3	0	0
CBA	4	-1	0
the Shapley values	$\frac{13}{6}$	$\frac{-5}{6}$	$\frac{10}{6}$

Core:

$$a \geq 0, b \geq -2, c \geq 0, a + b + c = 3, a + b \geq 0, a + c \geq 3, b + c \geq -1.$$

So (a,b,c) = (4, -2,1) belongs to the core.