Name: _____
SID: _____
Section: _____
Instructor: _____

MATH 230 FALL 2004 SECTION 4
MIDTERM EXAMINATION I
OCTOBER 5, 2004 6:30 - 7:45 PM

INSTRUCTIONS

- There are **10** problems on this exam for a total of **100** points. Some problems have multiple parts.
- PLEASE, SHOW YOUR WORK. ANSWERS WITHOUT SUPPORTING WORK WILL BE GIVEN NO CREDIT.
- Be sure your answers are legible and complete.
- You **may not** use CALCULATORS, BOOKS, or PERSONAL NOTES.
- **Do not** write on the line marked SCORE at the bottom of each page.
- Cellular phones must be turned off at the beginning of the exam.

1. (10 points) Show that the 4 points $P_1(0,0,0)$, $P_2(2,3,0)$, $P_3(1,-1,1)$, $P_4(1,4,-1)$ are coplanar (they lie on the same plane), and find the equation of the plane that contains them.

2. (10 points) Find the equation of the plane that is equidistant from the points $(3, 2, 1)$ and $(-3, -2, -1)$ (that is, every point on the plane has the same distance from the two given points).

3. (6 points) Find the vector projection of $\vec{b}$ onto $\vec{a}$, if $\vec{a} =< 4, 2, 0 >$ and $\vec{b} =< 1, 1, 1, >$.

4. (12 points) Consider the curve $\vec{r}(t) = \sqrt{2}\cos t\,\vec{i} + \sin t\,\vec{j} + \sin t\,\vec{k}$.

   (a) (8 points) Find the **unit tangent** vector function $\vec{T}(t)$ and the **unit normal** vector function $\vec{N}(t)$.

   (b) (4 points) Compute the curvature κ.

5. (10 points) Find the length of the curve with parametric equation:

$$\vec{r}(t) = < e^t, e^t \sin t, e^t \cos t >,$$

between the points $(1, 0, 1)$ and $(e^{2\pi}, 0, e^{2\pi})$.

6. (12 points) A spaceship is traveling with acceleration

$$\vec{a}(t) =< e^t, t, \sin 2t > .$$

At $t = 0$, the spaceship was at the origin, $\vec{r}(0) =< 0, 0, 0, >$, and had initial velocity $\vec{v}(0) =< 1, 0, 0 >$. Find the position of the spaceship at $t = \pi$.

7. (10 points) Write the equation of the tangent line to the curve with parametric equation

$$\vec{r}(t) =< \sqrt{t}, 1, t^4 >,$$

at the point $(1, 1, 1)$.

8. (12 points) Using cylindrical coordinates, find the parametric equations of the curve that is the intersection of the cylinder $x^2 + y^2 = 4$ and the cone $z = \sqrt{x^2 + y^2}$.

9. (6 points) Let $f(x, y) = \sin(x^2 + y^2) + \arcsin(y^2)$. Calculate:

$$\frac{\partial^2 f}{\partial x \partial y}.$$

10. (12 points) Show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{7x^2y(x-y)}{x^4+y^4}.$$

Justify your answer.