

1. (5 pts.) Differentiate $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$.

a) $y' = \frac{4}{(e^x + e^{-x})^2}$

b) $y' = -2 \frac{e^{2x} + e^{-2x}}{(e^x - e^{-x})^2}$

c) $y' = -\frac{2}{e^{2x} + e^{-2x}}$

d) $y' = -\frac{4}{(e^x - e^{-x})^2}$

e) $y' = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

2. (5 pts.) Find the inverse function for $y = e^{\sqrt[4]{x}}$.

a) $y = (\ln x)^4$

b) $y = 4 \ln x$

c) $y = \ln x^4$

d) $y = \ln 4x$

e) $y = e^{4x}$

3. (5 pts.) Differentiate $y = \frac{\ln x}{10 + x}$.

a) $y' = \frac{10 + x + x \ln x}{x(10 + x)^2}$

b) $y' = \frac{1}{x(10 + x)}$

c) $y' = \frac{10 + x - x \ln x}{(10 + x)^2}$

d) $y' = \frac{10 + x - x \ln x}{x(10 + x)^2}$

e) $y' = \frac{1}{10x}$

4. (5 pts.) Find the derivative of the function $y = 9 \sin^{-1}(x^2)$.

a) $y' = 18 \cos^{-1}(x)$

b) $y' = \frac{18x}{\sqrt{1 + x^4}}$

c) $y' = \frac{2x}{\sqrt{1 + x^4}}$

d) $y' = \frac{18x}{\sqrt{1 - x^2}}$

e) $y' = \frac{18x}{\sqrt{1 - x^4}}$

5. (5 pts.) Find $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 4x}$.

a) ∞

b) $-\infty$

c) -1

d) 0

e) $\frac{1}{4}$

6. (5 pts.) Evaluate $\int 12x \ln x \, dx$.

a) $6x^2 \ln(x) - 12x^2 + C$

b) $6x^2 \ln(x) - 3x + C$

c) $6x^2 \ln(x) - 3x^2 + C$

d) $6x^2 \ln(x) + 3x^2 + C$

e) $6x \ln(x) - 3x^2 + C$

7. (5 pts.) Evaluate $\int \sin^3 2x \, dx$.

a) $\frac{1}{6} \sin^3 2x - \frac{1}{2} \sin 2x + C$

b) $\frac{1}{2} \sin 2x - \frac{1}{4} \sin^3 2x + C$

c) $\frac{1}{6} \cos^3 2x - \frac{1}{2} \cos 2x + C$

d) $\frac{1}{2} \cos 2x - \frac{1}{6} \cos^3 2x + C$

e) $\frac{1}{2} \sin 2x - \frac{1}{6} \sin^3 2x + C$

8. (5 pts.) Evaluate $\int \frac{y}{y + 8} \, dy$.

a) $y - 16 \ln(y + 8) + C$

b) $y - 8 \ln|y + 8| + C$

c) $y + 8 \ln(y + 8) + C$

d) $y - 16 \ln|y + 8| + C$

e) $y + 8 \ln|y + 8| + C$

9. (5 pts.) Evaluate $\int_{-1}^{\frac{1}{\sqrt{3}}} \frac{e^{\arctan y}}{1 + y^2} \, dy$

a) $e^{-\pi/6} - e^{\pi/4}$

b) $e^{-\pi/6} + e^{\pi/4}$

c) 0

d) $e^{\pi/6} - e^{-\pi/4}$

e) 1

10. (5 pts.) Evaluate $\int_{-\infty}^0 \frac{6}{5x-2} dx$.
- 13
 - 6
 - The integral is divergent.
 - 7
 - 2
11. (5 pts.) If $y = Bt - 7, x = t - 2$ are the parametric equations of a curve and $y(x) = 8x + 9$ is the Cartesian equation of a same curve, find B .
- 8
 - 6
 - 6
 - 8
 - 9
12. (5 pts.) Find dy/dx where $x = 3te^t$ and $y = 3t + e^t$.
- $\frac{3 + e^t}{e^t(1+t)}$
 - $\frac{3t + e^t}{e^t(1+t)}$
 - $\frac{3t + e^t}{3e^t(1+t)}$
 - $\frac{3 + e^t}{3e^t(1+t)}$
 - $\frac{e^t}{3 + e^t}$
13. (5 pts.) Find the area A of the region that is bounded by the curve $r = \sqrt{\sin \theta}$ and lies in the sector $0 \leq \theta \leq 2\pi/3$.
- $A = \frac{1}{4}$
 - $A = \frac{3}{4}$
 - $A = \frac{1}{2}$
 - $A = 1$
 - $A = 2$
14. (5 pts.) Determine whether the sequence $\left\{ \frac{\ln(n)}{\ln(9n)} \right\}$ converges or diverges. If it converges, find the limit.
- converges to $\frac{1}{\ln(9)}$
 - converges to $\frac{1}{9}$
 - converges to 0
 - converges to 1
 - diverges
15. (5 pts.) Determine whether the sequence $\{n^8 e^{-n}\}$ converges or diverges. If it converges, find the limit.
- converges to 0
 - converges to 1
 - converges to $8e$
 - converges to $\frac{8}{e}$
 - diverges
16. (5 pts.) Determine whether the series $\sum_{n=1}^{\infty} 15 \left(\frac{5}{6}\right)^{n-1}$ is convergent or divergent. If it is convergent, find its sum.
- 90
 - 450
 - 75
 - 98
 - divergent
17. (5 pts.) Find values of r for which the series $\sum_{m=3}^{\infty} \frac{1}{m \ln m [\ln(\ln m)]^r}$ is convergent.
- $r \leq 1$
 - $r < 1$
 - $r > 1$
 - $r \geq 1$
 - $r = 1$

18. (5 pts.) Find the radius of convergence R and interval of convergence I of the series $\sum_{n=1}^{\infty} \frac{x^n}{2^n n^4}$.

a) $R = \frac{1}{2}, I = \left(-\frac{1}{2}, \frac{1}{2}\right]$

b) $R = 2, I = [-2, 2)$

c) $R = \frac{1}{2}, I = \left(-\frac{1}{2}, \frac{1}{2}\right)$

d) $R = 2, I = [-2, 2]$

e) $R = \frac{1}{2}, I = \left[-\frac{1}{2}, \frac{1}{2}\right)$

19. (5 pts.) Find a power series representation for the function $f(x) = \frac{1}{2 - x^3}$.

a) $\sum_{n=0}^{\infty} \frac{x^{3n}}{2^{n+1}}$

b) $\sum_{n=0}^{\infty} \frac{x^n}{2^{n+2}}$

c) $\sum_{n=0}^{\infty} -\frac{x^{3n}}{2^{n+1}}$

d) $\sum_{n=0}^{\infty} -\frac{x^{3n}}{2^n}$

e) $\sum_{n=0}^{\infty} (x^3 - 1)^n$

20. (5 pts.) Find the Maclaurin series for $f(x) = \cos(5x)$.

a) $\sum_{n=0}^{\infty} \frac{(-1)^n (5)^n x^{2n}}{(2n)!}$

b) $\sum_{n=0}^{\infty} \frac{(5)^{2n} x^{2n}}{(2n)!}$

c) $\sum_{n=0}^{\infty} \frac{(-1)^n (5)^{2n} x^{2n}}{(2n)!}$

d) $\sum_{n=0}^{\infty} \frac{(-1)^n (5)^{2n} x^{2n}}{n!}$

e) $\sum_{n=0}^{\infty} \frac{(-1)^n (5)^{2n} x^n}{(2n)!}$

For each series below, determine whether it is absolutely convergent, conditionally convergent, or divergent. **Code on your scantron sheet A** if the series is *Absolutely convergent*, **C** if it is *Conditionally convergent*, or **D** if it is *Divergent*.

21. (3 pts.) $\sum_{n=1}^{\infty} n \cos\left(\frac{1}{n}\right)$

26. (10 pts.) Find the area of the region that lies inside $r^2 = 4 \cos 2\theta$.

22. (3 pts.) $\sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n}$

23. (3 pts.) $\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$

24. (3 pts.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n}$

25. (3 pts.) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n}$

27. (10 pts.) Find $\int_{\sqrt{2}}^2 \frac{1}{t\sqrt{t^2-1}} dt$.

28. (15 pts.) Let $f(x) = \cos x$.

(a) Find $f(\pi)$, $f'(\pi)$, $f''(\pi)$, $f'''(\pi)$, $f^{(4)}(\pi)$, and $f^{(5)}(x)$.

$$f(x) =$$

$$f(\pi) =$$

$$f'(x) =$$

$$f'(\pi) =$$

$$f''(x) =$$

$$f''(\pi) =$$

$$f'''(x) =$$

$$f'''(\pi) =$$

$$f^{(4)}(x) =$$

$$f^{(4)}(\pi) =$$

$$f^{(5)}(x) =$$

(b) Use your answers in part (a) to write down the fourth degree Taylor polynomial for $f(x) = \cos x$ at $a = \pi$.

(c) Find the Taylor series for $f(x) = \cos x$ at $a = \pi$.

ITEM NO. FORM: A

1	D
2	A
3	D
4	E
5	E
6	C
7	C
8	B
9	D
10	C
11	A
12	D
13	B
14	D
15	A
16	A
17	C
18	D
19	A
20	C
21	D
22	D
23	A
24	A
25	C

26. 4

27. $\frac{\pi}{12}$

28.

a)

$$f(x) = \cos x; f'(x) = -\sin x; f''(x) = -\cos x; f'''(x) = \sin x; f^{(4)}(x) = \cos x; f^{(5)}(x) = -\sin x$$

$$f(\pi) = -1; f'(\pi) = 0; f''(\pi) = 1; f'''(\pi) = 0; f^{(4)}(\pi) = -1$$

b) $T_4(x) = -1 + \frac{1}{2}(x - \pi)^2 - \frac{1}{24}(x - \pi)^4$

c) $T(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n)!} (x - \pi)^{2n}$