1. Find $(f^{-1})'(3)$ for $f(x) = 3 + x^2 + \tan(\pi x/2)$.

a) $\frac{2}{\pi}$

b) $\frac{5}{\pi}$

c) $\frac{1}{\pi}$

d) $\frac{4}{\pi}$

e) $\frac{\pi}{2}$

2. Find the absolute maximum value of the function $g(x) = \frac{e^{-3x}}{x}$ for $-\infty < x < 0$.

a) $-2e$

b) $-3e$

c) $-4e$

d) $4e^{-2}$

e) $3e^{-3}$

3. Find the inverse function for $f(x) = e^{\sqrt[7]{x}}$.

a) $f^{-1}(x) = \ln 7x$

b) $f^{-1}(x) = \ln x^7$

c) $f^{-1}(x) = \frac{\ln x}{7}$

d) $f^{-1}(x) = (\ln x)^7$

e) $f^{-1}(x) = e^{x^7}$

4. Evaluate the integral $\int \frac{(\ln x)^2}{x} dx$.

a) $\frac{1}{3}(\ln x)^3 + C$

b) $-(\ln x)^3 + C$

c) $(\ln x)^3 + C$

d) $\frac{(\ln x)^3}{x^2} + C$

e) $-\frac{1}{3}(\ln x)^3 + C$

5. Find $\lim_{x \to 0^+} \frac{\ln 4x}{x}$.

a) 0

b) $\ln 4$

c) 4

d) π

e) $-\infty$

6. Evaluate the integral $\int 4x \ln x dx$.

a) $2x^2 \ln(x) - 4x^2 + c$

b) $2x \ln(x) - x^2 + C$

c) $2x^2 \ln(x) - x + C$

d) $2x^2 \ln(x) - x^2 + C$

e) $4x^2 \ln(x) + x^2 + C$

7. Evaluate the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot^2 x dx$.

a) $\sqrt{3} - \frac{\pi}{6}$

b) $\sqrt{3} - \frac{\pi}{3}$

c) $\frac{1}{\sqrt{3}} - \frac{\pi}{6}$

d) $\frac{1}{\sqrt{3}} + \frac{\pi}{6}$

e) $\frac{\pi}{2} - \frac{\pi}{3}$

8. Evaluate the integral $\int \frac{x+15}{(x+6)(x-3)} dx$.

a) $\ln\left|\frac{x+6}{(x-3)^2}\right| + C$

b) $\ln\left(\frac{x-3}{x+6}\right)^2 + C$

c) $\ln|(x-3)^2(x+6)| + C$

d) $\ln\left|\frac{(x-3)^2}{x+6}\right| + C$

e) $\ln\left|\frac{x+15}{(x+6)(x-3)}\right| + C$

9. Evaluate the integral $\int_{-\frac{1}{\sqrt{3}}}^{1} \frac{e^{\arctan y}}{1+y^2}\,dy$.

   a) $e^{\pi/4} + e^{-\pi/6}$

   b) $e^{-\pi/4} - e^{\pi/6}$

   c) 0

   d) $e^{\pi/4} - e^{-\pi/6}$

   e) $e^{\arctan(1)}$

10. Evaluate the improper integral $\int_0^{\infty} e^{-4x}\,dx$.

   a) 4

   b) $\frac{1}{4}$

   c) $-\frac{1}{4}$

   d) e^{-4}

   e) The integral diverges.

11. Find an equation of the tangent line to the curve $x = e^t, y = (t-6)^2$ at the point $(1, 36)$.

   a) $y = 12x + 50$

   b) $y = -12x + 54$

   c) $y = 12x + 47$

   d) $y = -12x + 48$

   e) $y = (x-6)^2$

12. Find a Cartesian equation for the curve describe by the polar equation $r = 9\sin\theta$.

   a) $x^2 + \left(y + \frac{9}{2}\right)^2 = \left(\frac{9}{2}\right)^2$

   b) $x^2 + \left(y - \frac{9}{2}\right)^2 = \left(\frac{9}{2}\right)^2$

   c) $\left(x - \frac{9}{2}\right)^2 + y^2 = \left(\frac{9}{2}\right)^2$

   d) $x^2 + \left(y + \frac{9}{2}\right)^2 = \left(\frac{9}{2}\right)$

   e) $x^2 + y^2 = 3^2$

13. Find the area of the region that is bounded by the curve $r = \sqrt{\sin\theta}$ and lies in the sector $0 \le \theta \le \pi/3$.

   a) $\frac{3}{4}$

   b) $\frac{1}{4}$

   c) 1

   d) $\frac{1}{2}$

   e) $\frac{\pi}{3}$

14. Determine whether the sequence $\left\{\frac{5^n}{7^{n+1}}\right\}$ converges or diverges. If it converges, find the limit.

   a) converges to $\frac{5}{7}$

   b) converges to 0

   c) converges to 1

   d) converges to $\frac{5}{49}$

   e) diverges

15. Find the value of the limit for the sequence $\left\{\arctan\left(\frac{8n}{8n+2}\right)\right\}$.

   a) 1

   b) $\frac{\pi}{2}$

   c) $\frac{1}{2}$

   d) $\frac{\pi}{4}$

   e) diverges

16. Determine whether the series $\sum_{n=1}^{\infty} 11\left(\frac{5}{6}\right)^{n-1}$ is convergent or divergent. If it is convergent, find its sum.

   a) 55

   b) 330

   c) 66

   d) 75

   e) divergent

17. Find the values of p for which the series $\sum_{n=1}^{\infty} n(1+n^2)^p$ is convergent.

a) $p = 0$

b) $p > 1$

c) $p < 1$

d) $p > -1$

e) $p < -1$

18. Find the radius of convergence R and interval of convergence I of the series $\sum_{n=1}^{\infty} (-1)^n \frac{(x+8)^n}{n4^n}$.

a) $R = 4$ and $I = [4, 12)$

b) $R = 1$ and $I = (7, 9)$

c) $R = 1$ and $I = [-8, 8]$

d) $R = 4$ and $I = (-12, -4]$

e) $R = 4$ and $I = [-8, 8]$

19. Evaluate the indefinite integral $f(x) = \int \frac{x}{1 - x^{10}} dx$ as a power series.

a) $C + \sum_{n=0}^{\infty} (-1)^n x^{10n+2}$

b) $C + \sum_{n=0}^{\infty} \frac{x^{10n+2}}{10n+2}$

c) $C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{10n+2}}{10n+2}$

d) $C + \sum_{n=0}^{\infty} \frac{x^{11n+1}}{11n}$

e) $C + \sum_{n=0}^{\infty} \frac{x^{11n+1}}{11n+1}$

20. If $f(x) = \sum_{n=0}^{\infty} b_n (x-10)^n$ for all x, write a formula for b_9.

a) $b_9 = \frac{f^{(9)}(10)}{9!}$

b) $b_9 = \frac{f^{(10)}(9)}{9!}$

c) $b_9 = \frac{f^{(10)}(9)}{10!}$

d) $b_9 = \frac{f^{(9)}(9)}{10!}$

e) $b_9 = \frac{f^{(9)}(10)}{9}$

For each series below, determine whether it is absolutely convergent, conditionally convergent, or divergent. **Code on your scantron sheet A** if the series is *Absolutely convergent*, **C** if it is *Conditionally convergent*, or **D** if it is *Divergent*.

21. (3pts.) $\sum_{n=1}^{\infty} \frac{(-5)^n}{3^{n+1}}$

22. (3pts.) $\sum_{n=1}^{\infty} (-1)^n n e^n$

23. (3pts.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$

24. (3pts.) $\sum_{n=1}^{\infty} \frac{n(-5)^n}{7^{n-1}}$

25. (3pts.) $\sum_{n=1}^{\infty} \left(\frac{3n^2+4}{4n^2+5}\right)^n$

For questions 26-29, you are given 5 equations, labeled a) through e), and 4 graphs, labeled I through IV. There is only one equation that corresponds to each graph. **Code your answers on the scantron sheet.**

a) $r = \sin 2\theta$

b) $r = \ln\theta, \theta \geq 1$

c) $r = 2(1 - \sin\theta)$

d) $x = e^{\theta}, y = e^{-\theta}$

e) $x = 2(\theta - \sin\theta), y = 2(1 - \cos\theta)$

26. (2pts.) Which equation corresponds to graph I?

27. (2pts.) Which equation corresponds to graph II?

28. (2pts.) Which equation corresponds to graph III?

29. 2pts.) Which equation corresponds to graph IV?

30. (12pts.) Find $\int \frac{dx}{x^2\sqrt{16x^2 - 9}}$.

31. (15pts.) Let $f(x) = \cos x$.

(a) Find $f\left(\frac{\pi}{3}\right), f'\left(\frac{\pi}{3}\right), f''\left(\frac{\pi}{3}\right), f'''\left(\frac{\pi}{3}\right), f^{(4)}\left(\frac{\pi}{3}\right)$, and $f^{(5)}(x)$.

(b) Use your answer in part a) to write down the fourth degree Taylor polynomial for $f(x) = \cos x$ at $a = \frac{\pi}{3}$.

(c) Use Taylor's inequality to estimate the accuracy of approximation of this fourth degree Taylor polynomial of $\cos(x)$ when $0 \leq x \leq \frac{2\pi}{3}$.