1. Find $\lim_{x \to 0^+} \frac{\ln(1+2x)}{x}$.

a) 0

b) 2

c) $+\infty$

d) $-\infty$

e) 1

2. Compute the SUM of the following convergent series:

$$1 - \frac{e}{\pi} + \frac{e^2}{\pi^2} - \frac{e^3}{\pi^3} + \dots$$

a) $e + \pi$

b) 1

c) $e - \pi$

d) $\frac{\pi}{\pi - e}$

e) $\frac{\pi}{\pi + e}$

3. Determine which one of the following integrals is improper:

a) $\int_{-1}^{1} \frac{1}{2+x^2}\, dx$

b) $\int_{-1}^{1} \sec(x)\, dx$

c) $\int_{-1}^{1} \frac{2}{4x^2-1}\, dx$

d) $\int_{-1}^{1} \ln(1+x^2)\, dx$

e) None of the above.

4. Determine whether the given sequence converges or diverges. If it converges, calculate its limit:

$$a_n = \frac{\sin n}{n^4}.$$

a) converges to 1

b) converges to 0

c) converges to -1

d) diverges

e) converges to π

5. Evaluate the integral $\int_{-1}^{0} \frac{x}{x+1}\, dx$.

a) 0

b) 2

c) 1

d) It diverges

e) None of the above

6. If the series $\sum_{n=1}^{+\infty} a_n$ converges, then $\lim_{n \to +\infty} (a_n + 1)$ is:

a) 1

b) 0

c) 2

d) does not exist

e) e

7. Find $\lim_{x \to +\infty} x \sin\left(\frac{1}{x}\right)$.

a) -1

b) $+\infty$

c) 0

d) 1

e) $-\infty$

8. For each of the series (1) and (2) given below choose the right answer:

$$(1) \quad \sum_{n=1}^{+\infty} \frac{4+(-1)^n}{n^4}, \qquad (2) \quad \sum_{n=1}^{+\infty} \frac{2+3^n}{2^n}.$$

a) Only (1) converges.

b) Only (2) converges.

c) Both diverge.

d) Both converge.

e) None of the above.

9. Given the series $\sum_{n=1}^{+\infty} \frac{\cos(n\pi)}{n}$, which of the following statements are true?

(i) The series diverges by comparison with $\sum_{n=1}^{+\infty} \frac{1}{n}$.
(ii) The series is a geometric series.
(iii) The series converges by the Alternating Series Test.

a) Only (i) is true.

b) Only (ii) is true.

c) Only (iii) is true.

d) Only (ii) and (iii) are true.

e) None are true.

For Problems 10–14, determine whether each series is absolutely convergent, conditionally convergent, or divergent. **Code on your scantron sheet: A** if the series is *Absolutely convergent*, **C** if it is *Conditionally Convergent*, **D** if it is *Divergent*.

10. $\sum_{n=1}^{+\infty} \frac{n\cos(1/n)}{2n+1}$

11. $\sum_{n=1}^{+\infty} (-1)^n \frac{5^{n+1}}{n^6}$

12. $\sum_{n=1}^{+\infty} (-1)^{n+1} \left(\frac{1}{3+7n^2}\right)^{2n}$

13. $\sum_{n=1}^{+\infty} \frac{(-1)^n}{1+\sqrt{n}+n}$

14. $\sum_{n=1}^{+\infty} (\sqrt[n]{3}-1)^n$

15. (10 pts.) Determine whether the following integral is convergent or divergent. If convergent, find its value:

$$\int_1^{+\infty} x\, e^{-x}\, dx.$$

Justify each step carefully.

16. (10 pts.) Determine whether the following series is convergent or divergent:

$$\sum_{n=1}^{+\infty} \frac{1+\sin(1/n)}{n^3+1}.$$

Justify your answer carefully.

17. (10 pts.) Use the Integral Test to determine whether the following series is convergent or divergent:

$$\sum_{n=2}^{+\infty} \frac{1}{n(\ln n)^2}.$$

15. (10 pts.) Determine whether the following integral is convergent or divergent. If convergent, find its value:

$$\int_1^{+\infty} x e^{-x}\, dx.$$

Justify each step carefully.

Improper Integral of type II :

$$I = \int_1^{+\infty} x e^{-x} dx = \lim_{t \to +\infty} \int_1^t x e^{-x} dx$$

$$\int x e^{-x} dx = -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x} + C$$

by parts with

$u = x \quad v' = e^{-x} \Rightarrow v = -e^{-x}$

$$\Rightarrow I = \lim_{t \to +\infty} \left[-te^{-t} - e^{-t} + 2e^{-1} \right] = 2e^{-1} - \lim_{t \to +\infty} te^{-t}$$

($e^{-t} \to 0$ as $t \to +\infty$)

$L = \lim_{t \to +\infty} te^{-t}$ indeterminate form of type $0 \cdot \infty$

by L'Hôpital's Rule :

$$L = \lim_{t \to +\infty} \frac{t}{e^t} \overset{H}{=} \lim_{t \to +\infty} \frac{1}{e^t} = 0$$

$$\Rightarrow \boxed{I = \frac{2}{e}} \quad \text{convergent.}$$

EXAM II- FORM A

1. B	10. D
2. E	11. D
3. C	12. A
4. B	13. C
5. D	14. A
6. A	
7. D	
8. A	
9. C	

16. (10 pts.) Determine whether the following series is convergent or divergent:

$$\sum_{n=1}^{+\infty} \frac{1 + \sin(1/n)}{n^3 + 1}.$$

Justify your answer carefully.

Since $\frac{1}{n} \leq 1 \ (n \geq 1)$, $0 < \sin\left(\frac{1}{n}\right) < 1$, so series has strictly positive terms.

We can apply comparison test with $\sum_{n=1}^{+\infty} \frac{2}{n^3}$

Since :

$$\frac{1 + \sin(1/n)}{n^3 + 1} < \frac{2}{n^3 + 1} < \frac{2}{n^3} \quad \text{for all } n \geq 1$$

Since $\sum_{n=1}^{+\infty} \frac{2}{n^3} = 2 \sum_{n=1}^{+\infty} \frac{1}{n^3}$ converges as a p-series with $p = 3 > 1$, $\sum_{n=1}^{+\infty} \frac{1 + \sin\left(\frac{1}{n}\right)}{n^3 + 1}$ also converges.

the Limit comparison test can also be used

17. (10 pts.) Use the Integral Test to determine whether the following series is convergent or divergent:

$$\sum_{n=2}^{+\infty} \frac{1}{n(\ln n)^2}.$$

Determine first whether Integral test applies :

$$\sum_{n=2}^{+\infty} \frac{1}{n(\ln n)^2} = \sum_{n=2}^{+\infty} f(n) \quad \text{with } f(x) = \frac{1}{x \ln(x)^2} \quad x \geq 2$$

$\Rightarrow f$ continuous and positive, since $\ln x > 0$ for $x \geq 2$

and $f'(x) = \frac{-1}{(x(\ln x)^2)^2} ((\ln x)^2 + 2 \ln x) < 0$ for $x \geq 2$

$\Rightarrow f$ decreasing. (could use use [illegible] show $x_1 < x_2$ [illegible])

By Integral test, series converges $\Longleftrightarrow$

$\int_2^{+\infty} \frac{1}{x(\ln x)^2} dx$ converges :) may [illegible] with [illegible]

$$\int_2^{+\infty} \frac{1}{x(\ln x)^2} dx = \lim_{t \to +\infty} \int_2^t \frac{1}{x(\ln x)^2} dx = \lim_{t \to +\infty} \left[-(\ln x)^{-1} \right]_2^t$$

$u = \ln x$

$$= \frac{1}{\ln 2} - \lim_{t \to +\infty} \frac{1}{\ln t} = \frac{1}{\ln 2}$$

So series converges.