

1. Differentiate the function  $y = e^{ax^5}$  where  $a > 0$  is a constant.

a)  $y' = 4ax^4e^{ax^5}$

b)  $y' = e^{ax^5}$

c)  $y' = 5ax^4e^{ax^5}$

d)  $y' = ax^4e^{ax^5}$

e)  $y' = 5e^{ax^5}$

2. Solve the equation  $e^{9x+2} = k$  for  $x$ .

a)  $x = \frac{\ln(k) + 2}{9}$

b)  $x = \frac{\sqrt[9]{k} - 2}{9}$

c)  $x = \frac{\ln(k) - 2}{9}$

d)  $x = \frac{\sqrt[9]{k} + 2}{9}$

e)  $x = \ln\left(\frac{k-2}{9}\right)$

3. Differentiate the function  $f(x) = \cos(\ln(5x))$ .

a)  $f'(x) = -\frac{\sin(\ln(5x))}{x}$

b)  $f'(x) = -\sin(\ln(5x))$

c)  $f'(x) = \frac{1}{\cos(\ln(5x))}$

d)  $f'(x) = \frac{\sin(\ln(5x))}{x}$

e)  $f'(x) = -\sin\left(\frac{1}{x}\right)$

4. Evaluate the integral  $\int \frac{\ln x}{x} dx$ .

a)  $\frac{1}{2}(\ln x)^2 + C$

b)  $-\frac{1}{2}(\ln x)^2 + C$

c) 0

d)  $(\ln x)^2 + C$

e)  $-(\ln x)^2 + C$

5. Find  $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 3x}$ .

a)  $\infty$

b) 0

c)  $-\infty$

d)  $\frac{1}{3}$

e)  $-9$

6. Evaluate the integral  $\int x^7 \ln(x) dx$ .

a)  $\frac{1}{7}x^8 \ln(x) - \frac{1}{49}x^8 + C$

b)  $\frac{1}{8}x^8 \ln(x) - \frac{1}{8}x^8 + C$

c)  $\frac{1}{8}x^8 \ln(x) - \frac{1}{64}x^8 + C$

d)  $\frac{1}{8}x^8 \ln(x) + \frac{1}{64}x^8 + C$

e) none of these

7. Evaluate the integral  $\frac{1}{2\pi} \int_{-\pi}^{\pi} \sin x \cos^3 x dx$ .

a)  $\frac{\pi}{5}$

b) 0

c)  $\frac{\pi}{2}$

d)  $\pi$

e) 1

8. Evaluate the integral  $\int_{\sqrt{2}}^2 \frac{1}{t^2 \sqrt{t^2 - 1}} dt$ .

a)  $\frac{\pi}{4}$

b)  $\frac{1}{4}$

c)  $\frac{\sqrt{3} - 1}{2}$

d)  $\frac{\pi}{12}$

e)  $\frac{\sqrt{3 - \sqrt{2}}}{2}$

9. Write the form of the partial fraction decomposition of the expression  $\frac{5}{(4x^2 + 1)(x - 8)^2}$ .

- a)  $\frac{A}{4x^2} + \frac{B}{1} + \frac{C}{x^2} - \frac{D}{8}$
- b)  $\frac{A}{4x^2 + 1} + \frac{B}{(x - 8)^2}$
- c)  $\frac{Ax + B}{4x^2 + 1} + \frac{Cx + D}{(x - 8)^2}$
- d)  $\frac{A}{4x^2} + \frac{B}{1} + \frac{C}{x^2} + \frac{D}{8}$
- e)  $\frac{Ax + B}{4x^2 + 1} + \frac{C}{x - 8} + \frac{D}{(x - 8)^2}$

10. Which of the following integrals is improper?

- a)  $\int_1^2 3x^2 e^{-x^2} dx$
- b)  $\int_{-6}^0 \frac{1}{x^2 + 6} dx$
- c)  $\int_0^4 \frac{x}{x^2 - 7x + 10} dx$
- d)  $\int_{13}^{15} \frac{x - 4}{x - 7} dx$
- e)  $\int_0^\pi \sin x dx$

11. Find the slope of the tangent line to the curve  $x = 2te^t, y = 2t + e^t$  at the point  $t = 0$ .

- a) 3
- b)  $\frac{3}{2}$
- c)  $\frac{1}{2}$
- d) 0
- e) 1

12. Find a polar equation for the curve represented by the Cartesian equation  $x^2 = 6y$ .

- a)  $r = 6 \tan \theta$
- b)  $r = 6 \tan \theta \sec \theta$
- c)  $r = 6 \tan \theta \sin \theta$
- d)  $r = 6 \cos \theta \sin \theta$
- e)  $r = 6 \sin \theta$

13. Find the area of the region enclosed by one loop of the curve  $r = 2 \cos 4\theta$ .

- a)  $\frac{\pi}{11}$
- b)  $\pi$
- c)  $4\pi$
- d)  $\frac{\pi}{4}$
- e)  $2\pi$

14. Determine whether the sequence  $\left\{ \frac{8^n}{5^{n+1}} \right\}$  converges or diverges. If it converges, find the limit.

- a) converges to  $\frac{5}{8}$
- b) converges to 0
- c) converges to 1
- d) converges to  $\frac{5}{64}$
- e) diverges

15. Determine whether the sequence  $\{n^8 e^{-n}\}$  converges or diverges. If it converges, find the limit.

- a) converges to  $\frac{8}{e}$
- b) converges to 0
- c) converges to  $8e$
- d) converges to 1
- e) diverges

16. Determine whether the series  $\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$  is convergent or divergent. If it is convergent, find its sum.

- a)  $\frac{3}{2}$
- b)  $\frac{2}{3}$
- c) 1
- d)  $\frac{1}{2}$
- e) divergent

17. What is the minimum number of terms of the series  $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$  that must be summed in order to approximate the series so that  $|error| < 0.5$ ?

- a) 6
- b) 4
- c) 3
- d) 1
- e) 2

18. Find the radius of convergence and interval of convergence of the series  $\sum_{n=1}^{\infty} \frac{x^n}{4^n n^2}$ .

- a)  $R = \frac{1}{4}, I = \left(-\frac{1}{4}, \frac{1}{4}\right]$
- b)  $R = \frac{1}{4}, I = \left(-\frac{1}{4}, \frac{1}{4}\right)$
- c)  $R = 4, I = [-4, 4]$
- d)  $R = 4, I = [-4, 4)$
- e)  $R = \frac{1}{4}, I = \left[-\frac{1}{4}, \frac{1}{4}\right)$

19. Find a power series representation for the function  $f(z) = \frac{1}{6 - z^3}$ .

- a)  $\sum_{n=0}^{\infty} -\frac{z^{3n}}{6^n}$
- b)  $\sum_{n=0}^{\infty} \frac{z^n}{6^{n+2}}$
- c)  $\sum_{n=0}^{\infty} \frac{z^{3n}}{6^{n+1}}$
- d)  $\sum_{n=0}^{\infty} -\frac{z^{3n}}{6^{n+1}}$
- e)  $\sum_{n=0}^{\infty} (z^3 - 5)^n$

20. Find the Taylor series for  $f(x) = \ln x$  centered at  $a = 1$ .

- a)  $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n}$
- b)  $\sum_{n=2}^{\infty} \frac{(-1)^{n-1} (x-1)^n}{n!}$
- c)  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} (x+1)^n}{n}$
- d)  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} (x-1)^n}{n}$
- e)  $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n!}$

For each series below, determine whether it is absolutely convergent, conditionally convergent, or divergent. **Code on your scantron sheet A** if the series is *Absolutely convergent*, **C** if it is *Conditionally convergent*, or **D** if it is *Divergent*.

- 21. (3 pts.)  $\sum_{n=1}^{\infty} \frac{(-4)^{n+1}}{5^n}$
- 22. (3 pts.)  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+5}$
- 23. (3 pts.)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$
- 24. (3 pts.)  $\sum_{n=1}^{\infty} \frac{(-3)^n}{n4^{n+1}}$
- 25. (3 pts.)  $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2n+1}{3n+2}\right)^n$

For questions 26-29, you are given 5 equations, labeled a) through e), and 4 graphs, labeled I through IV. There is only one equation that corresponds to each graph. **Code your answers on the scantron sheet.**

- a)  $r = 2 \cos \theta$
- b)  $r = \cos 2\theta$
- c)  $r = 1 - 2 \cos \theta$
- d)  $x = \cos \theta, y = \sin 2\theta$
- e)  $x = \cos \theta, y = 1 - \cos^2 \theta$

- 26. (2 pts.) Which equation corresponds to graph I?
- 27. (2 pts.) Which equation corresponds to graph II?
- 28. (2 pts.) Which equation corresponds to graph III?

29. (2 pts.) Which equation corresponds to graph IV?

30. (12 pts.) Find  $\int \frac{1-x}{x(x^2+1)} dx$ .

31. (15 pts.) Let  $f(x) = e^{-3x}$ .

- a) (a) Find  $f(0)$ ,  $f'(0)$ ,  $f''(0)$ ,  $f'''(0)$ ,  $f^{(4)}(0)$  and  $f^{(5)}(0)$ .
- b) (b) Use your answers in part (a) to determine the Maclaurin series for  $f(x)$  and write the series in summation notation.
- c) (c) Find the radius of convergence for the Maclaurin series found in part (b) .