

1. Find  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{5x^2}$ .
- 5
  - $\infty$
  - 0
  - $-\infty$
  - $\frac{1}{10}$
2. Find  $\lim_{x \rightarrow -\infty} x^9 e^x$ .
- 0
  - 9
  - $\infty$
  - $\frac{1}{9}$
  - $-\infty$
3. Evaluate the integral  $I = \int_0^{\infty} e^{-2x} dx$ .
- $I = 2$
  - $I = \frac{1}{2}$
  - $I = -\frac{1}{2}$
  - The integral is divergent.
  - $I = -2$
4. Which of the following integrals is convergent?
- $\int_0^7 4 \frac{dx}{\sqrt{x}}$
  - $\int_{-7}^0 2 \frac{dx}{x^2}$
  - $\int_0^7 \frac{5}{6} \frac{dx}{x\sqrt{x}}$
  - $\int_0^7 \frac{7}{x} dx$
  - All of these integrals are divergent.
5. Determine whether the sequence  $\left\{ \frac{5^n}{6^{n+1}} \right\}$  converges or diverges. If it converges, find the limit.
- converges; limit = 1
  - converges; limit =  $\frac{5}{36}$
  - converges; limit = 0
  - converges; limit =  $\frac{5}{6}$
  - diverges
6. Determine whether the series  $\sum_{n=1}^{\infty} 9^{-n} 10^{n+1}$  is convergent or divergent. If it is convergent, find its sum.
- 76
  - 106
  - 60
  - 90
  - divergent
7. Find the values of  $p$  for which the series  $\sum_{m=5}^{\infty} \frac{1}{m \ln m [\ln(\ln m)]^p}$  is convergent.
- $p \leq 1$ .
  - $p < 1$ .
  - $p = 1$ .
  - $p > 1$ .
  - $p \geq 1$ .
8. Determine whether the series  $\sum_{k=1}^{\infty} \frac{8 + \cos k}{3^k}$  converges or diverges.
- The series is convergent by the comparison test.
  - The series is divergent by the comparison test.
  - The series is convergent by the integral test.
  - The series is convergent by the ratio test.
  - The series is divergent by the  $p$ -series test.

9. What minimum number of terms of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$  do we need to add in order to find the sum with ( $|error| < 0.0003$ )?

- a) 2
- b) 11
- c) 10
- d) 7
- e) 4

10. The terms of a series are defined recursively by the equations  $a_1 = 2$  and  $a_{n+1} = \frac{7n+1}{8n+7}a_n$ . Determine whether  $\sum a_n$  converges or diverges.

- a) diverges by comparison test
- b) diverges by ratio test
- c) converges by ratio test
- d) converges by alternating series test
- e) diverges by root test

11. For which of the following series is the ratio test inconclusive (that is, it fails to give a definite answer)?

- a)  $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$
- b)  $\sum_{n=1}^{\infty} \frac{(-5)^{n-1}}{\sqrt{n}}$
- c)  $\sum_{n=1}^{\infty} \frac{n(-3)^n}{8^{n-1}}$
- d)  $\sum_{n=1}^{\infty} \frac{1}{n^3}$
- e)  $\sum_{n=1}^{\infty} \frac{n}{3^n}$

12. Test the series  $\sum_{n=1}^{\infty} (-1)^n \frac{3^n n^6}{n!}$  for convergence or divergence.

- a) The series is divergent by ratio test.
- b) The series is conditionally convergent by ratio test.
- c) The series is absolutely convergent by ratio test.
- d) The series is divergent by integral test.
- e) The series is conditionally convergent by root test.

13. (10 pts.) Determine the convergence or divergence of the improper integral

$$\int_0^3 x^2 \ln(x) dx.$$

Explain the reason if it diverges and evaluate the integral if it converges.

14. (15 pts.) Determine whether the following sequences converge or diverge. If diverge, explain the reason and if converge, find the limit.

- a)  $\left\{ \frac{n \sin(n)}{n^3 + 1} \right\}$
- b)  $\left\{ \frac{3 + n^4}{4 + 3n^2} \right\}$
- c)  $\left\{ \frac{\ln(5n)}{\ln(n)} \right\}$
- d)  $\left\{ \frac{4^n}{n!} \right\}$
- e)  $\left\{ \frac{(-9)^{n+1}}{10^n} \right\}$

15. (15 pts.) Determine whether the following converge absolutely, converge conditionally or diverge and circle the correct answer. For each series, list the type of series or test(s) you used to justify your answer.

- a)  $\sum_{n=1}^{\infty} \left[ \frac{1}{n} - \frac{1}{n+1} \right]$
- b)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$
- c)  $\sum_{n=1}^{\infty} \frac{n(-4)^n}{5^{n-1}}$
- d)  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{4n^2 + 3}{4 + 3n^2}$
- e)  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(\ln n)^n}$