

1. Find all critical numbers of  $h(x) = \frac{x^2}{x-3}$ .
- 0, 2
  - 0, 6
  - 0, 2, 3
  - 0, 3, 6
2. What is the absolute maximum value of the function  $f(x) = x + \sin x$  on the interval  $[0, \frac{3\pi}{2}]$ ?
- $\pi$
  - $\frac{3\pi}{2}$
  - $\frac{3\pi}{2} - 1$
  - $\frac{3\pi}{2} + 2$
3. Suppose  $f(x) = \sqrt{3x+1}$ . Find the point  $x = c$  where the conclusion of the Mean Value Theorem is satisfied on the interval  $[1, 5]$ .
- 0
  - $\frac{3}{2}$
  - 4
  - $\frac{8}{3}$
4. Suppose  $h(1) = 6$  and  $-2 \leq h'(x) \leq 4$  for all real numbers. What is the largest possible value of  $h(6)$ ?
- 11
  - 21
  - 26
  - $\infty$
5. Suppose  $f$  is a differentiable function with derivative  $f'(x) = x^3(x-1)^2(x-3)$ . Where does  $f(x)$  have local extrema?
- Local maximum at  $x = 0$ ; local minimum at  $x = 3$
  - Local maxima at  $x = 0$  and  $x = 3$ ; local minimum at  $x = 1$
  - Local maximum at  $x = 3$ ; local minimum at  $x = 0$
  - Local maximum at  $x = 1$ ; local minimum at  $x = 0$  and  $x = 3$
6. Suppose  $f''(x) = (2-x)(x-4)^2$ . On what interval(s) is the graph of  $y = f(x)$  concave downward?
- $(2, 4)$  and  $(4, \infty)$
  - $(-\infty, 2)$
  - $(2, 4)$
  - $(-\infty, 2)$  and  $(4, \infty)$
7. Find all asymptotes of  $f(x) = \frac{(x+1)^2}{x^2-2x}$ .
- $x = 2, y = 0$
  - $x = 0, y = -1$
  - $x = 0, x = 2, y = 0$
  - $x = 0, x = 2, y = 1$
8. Determine  $\lim_{x \rightarrow \infty} \frac{(1+x+x^4)^{\frac{1}{2}}}{x^2}$ .
- 0
  - 1
  - $-\infty$
  - $\infty$
9. A farmer wants to enclose a rectangular area with 400 feet of fence. In order to maximize the area within the fence what function do we wish to maximize?
- $A(x) = 400x - x^2$
  - $A(x) = 200x - x^2$
  - $A(x) = 200 - 2x$
  - $A(x) = 400x^2$
10. Find the equation of the slant asymptote of the graph of  $f(x) = \frac{2x^2 - 5x}{x-1}$ .
- $y = 2x + \frac{3}{x-1}$
  - $y = 2x - 7$
  - $y = 2x - 3$
  - $y = 2x - 1$

11. Given  $f''(x) = x^3 - 2x + 1$ ,  $f'(0) = 1$ , and  $f(0) = 0$ , which one of the following functions represents  $f(x)$ ?
- a)  $f(x) = x^5 - 2x^3 + x$
  - b)  $f(x) = 20x^5 - 6x^3 + 2x^2$
  - c)  $f(x) = \frac{5}{4}x^5 - 2x^3 + 1$
  - d)  $f(x) = \frac{1}{20}x^5 - \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$
12. Which one function  $f$  is the antiderivative of  $F(x) = -\cos x - \sec^2 x$ ?
- a)  $f(x) = -\sin x - \tan x$
  - b)  $f(x) = \sin x + \tan x$
  - c)  $f(x) = \sin x - \tan^2 x$
  - d)  $f(x) = -\sin x + \tan^2 x$
13. A particle located at the origin when  $t = 0$  moves along the  $x$ -axis with velocity  $v(t) = \frac{1}{2}t^2 - t$  ft/s. Let  $s(t)$  be its position at time  $t$ . Which one of the following represents  $s(t)$ ?
- a)  $s(t) = t - 1$
  - b)  $s(t) = \frac{1}{6}t^3 - \frac{1}{2}t^2$
  - c)  $s(t) = \frac{3}{2}t^3 - 2t^2$
  - d)  $s(t) = \frac{1}{2}t^3 - t^2$

True/False. Bubble a for true, b for false.

14. If  $f$  is differentiable in  $[a, b]$ , then  $f$  will always have an absolute minimum point in  $[a, b]$ .

- a) True
- b) False

19. (15 points) Given  $f(x) = \frac{x}{1-x^2}$ ,  $f'(x) = \frac{1+x^2}{(1-x^2)^2}$ ,  $f''(x) = \frac{2x(x^2+3)}{(1-x^2)^3}$ , provide the information requested. Write "none" where appropriate.

- $x$ -intercept(s)= \_\_\_\_\_
- $y$ -intercept= \_\_\_\_\_
- vertical asymptote(s): \_\_\_\_\_
- horizontal asymptote: \_\_\_\_\_
- Interval(s) where  $f(x)$  is increasing: \_\_\_\_\_
- Interval(s) where  $f(x)$  is decreasing: \_\_\_\_\_
- Local minimum point(s),  $(x,y)$ : \_\_\_\_\_
- Local maximum point(s),  $(x,y)$ : \_\_\_\_\_
- Interval(s) where  $f(x)$  is concave up: \_\_\_\_\_
- Interval(s) where  $f(x)$  is concave down: \_\_\_\_\_
- $x$ -coordinate(s) of inflection point(s) for  $f(x)$ : \_\_\_\_\_
- Sketch a graph of  $f(x)$ .

15. If  $f''(c) = 0$ , then  $f(x)$  has an inflection point at  $x = c$ .

- a) True
- b) False

16. Given  $f(x) = \frac{g(x)}{h(x)}$ . If  $h(3) = 0$ , then  $x = 3$  must always be a vertical asymptote.

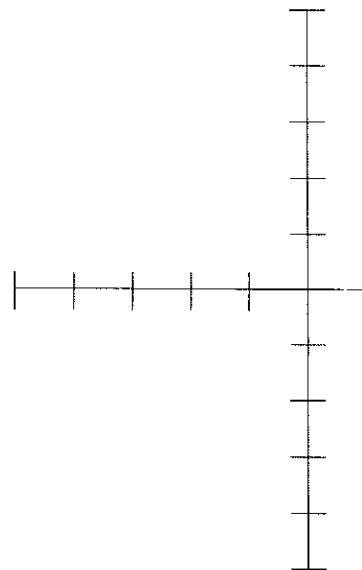
- a) True
- b) False

17. If  $f$  has an absolute minimum value at  $c$ , then  $f'(c) = 0$

- a) True
- b) False

18. If  $f'(x)$  exists and is nonzero for all  $x$ , then  $f(1) \neq f(0)$ .

- a) True
- b) False



20. (10 points) A rectangular box with an open top is to be constructed using 54 square inches of material. The length  $l$  is to be twice the width  $w$ . Find the dimensions that will maximize the volume of the box. You must justify that your answer is a maximum.

$$l = \underline{\hspace{2cm}}$$

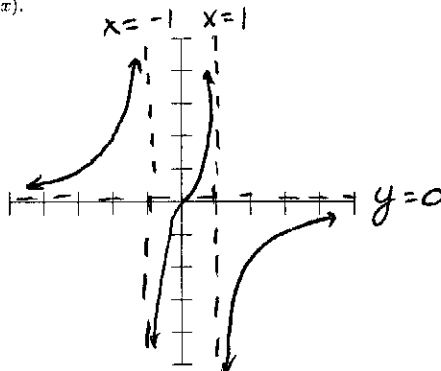
$$w = \underline{\hspace{2cm}}$$

$$h = \underline{\hspace{2cm}}$$

- 1. B
- 2. C
- 3. D
- 4. C
- 5. A
- 6. A
- 7. D
- 8. B
- 9. B
- 10. C
- 11. D
- 12. A
- 13. B

19. (15 points) Given  $f(x) = \frac{x}{1-x^2}$ ,  $f'(x) = \frac{1+x^2}{(1-x^2)^2}$ ,  $f''(x) = \frac{2x(x^2+3)}{(1-x^2)^3}$ , provide the information requested. Write "none" where appropriate.

- x-intercept(s) = 0
- y-intercept = 0
- vertical asymptote(s):  $x=1, x=-1$
- horizontal asymptote:  $y=0$
- Interval(s) where  $f(x)$  is increasing:  $(-\infty, -1), (-1, 1), (1, \infty)$
- Interval(s) where  $f(x)$  is decreasing: none
- Local minimum point(s), (x,y): none
- Local maximum point(s), (x,y): none
- Interval(s) where  $f(x)$  is concave up:  $(-\infty, -1), (0, 1)$
- Interval(s) where  $f(x)$  is concave down:  $(-1, 0), (1, \infty)$
- x-coordinate(s) of inflection point(s) for  $f(x)$ :  $x=0$
- Sketch a graph of  $f(x)$ .



20.  $l = 6''$   
 $w = 3''$   
 $h = 2''$