1. Compute the average rate of change $\left(\frac{\Delta y}{\Delta x}\right)$ for the function $f(x) = \sin x$ on the interval $\left[0, \frac{\pi}{4}\right]$.

   a) $\frac{8}{\pi}$

   b) $\frac{\sqrt{2}}{\pi}$

   c) $\frac{14}{\pi}$

   d) $\frac{\pi}{4\sqrt{2}}$

   e) $\frac{2\sqrt{2}}{\pi}$

2. If $f(x) = \left(x - \frac{1}{x}\right)^2$, find $f'(2)$.

   a) 3

   b) $\frac{9}{4}$

   c) $\frac{15}{8}$

   d) $\frac{5}{2}$

   e) $\frac{15}{4}$

3. If $x^2y + y^3 = 10$, find $\frac{dy}{dx}$ at the point $(1, 2)$.

   a) $-\frac{4}{13}$

   b) 0

   c) $-\frac{1}{3}$

   d) -6

   e) $\frac{dy}{dx}$ is undefined at the point $(1, 2)$.

4. The height of a triangle is increasing at a rate of 4 cm/min. The base is decreasing at a rate of 5 cm/min. At what rate is the area changing when the height measures 10 cm, and the base measures 15 cm?

   a) 1500 cm^2/min

   b) 5 cm^2/min

   c) -1 cm^2/min

   d) -10 cm^2/min

   e) -20 cm^2/min

5. If $f(x) = \frac{x^2 + 7}{x + 3}$, find all critical numbers of $f(x)$.

   a) $-3, -\sqrt{7}, \sqrt{7}$

   b) $-\sqrt{7}, \sqrt{7}$ only

   c) $-7, -3, 1$

   d) $-7, 1$ only

   e) -3 only

6. A particle moves along a straight line path with velocity given by $v(t) = 3t^2 - 4t + 5$. Find its position at $t = 2$ if its position at $t = 1$ is 6.

   a) 0

   b) 2

   c) 3

   d) 6

   e) 12

7. A box with a square base and no top is to have a volume of 32 cm^3. Find the least amount of material (surface area) needed to construct such a box.

   a) 64 cm^2

   b) 48 cm^2

   c) 32 cm^2

   d) 16 cm^2

   e) 128 cm^2

For each of the statements 8–11 below, indicate which of the following theorems is illustrated. Bubble the corresponding letter on your scantron.

a. Intermediate Value Theorem
b. Squeeze Theorem
c. Mean Value Theorem
d. Extreme Value Theorem
e. None of these theorems applies to this statement.

8. If $f(x) = 3x^3 - x - 1$, then there exists at least one solution of $f(x) = 0$ on $(0, 1)$.

(Bubble one of a–e.)

9. If it takes you three hours to drive the 150 miles from State College to Pittsburgh, then at some point on the trip your velocity is exactly 50 mph.

(Bubble one of a–e.)

10. Since $-1 \le \sin\dfrac{1}{x} \le 1$ for all $x \neq 0$,
then it is true that $-|x| \le |x| \sin\dfrac{1}{x} \le |x|$,
and so we can prove $\lim\limits_{x\to 0} x \sin\dfrac{1}{x} = 0$.

(Bubble one of a–e.)

11. If $f(x) = |3x - 4|$, there exists an absolute minimum of $f(x)$ on the interval $[2, 6]$.

(Bubble one of a–e.)

12. Evaluate the Riemann sum for $f(x) = x^2 - x$ if $0 \le x \le 2$ with four equal subintervals using right-hand endpoints as the sample points.

a) $\dfrac{1}{4}$

b) $\dfrac{1}{3}$

c) $\dfrac{5}{4}$

d) $\dfrac{2}{3}$

e) 1

13. Evaluate $\displaystyle\int_1^4 \frac{x^{3/2} + 4}{x^2}\ dx$.

a) 2

b) $-\dfrac{1}{2}$

c) 5

d) $\dfrac{119}{16}$

e) 1

14. Find the derivative of the function $G(x) = \displaystyle\int_\pi^{x^2} \sqrt{\sin t}\ dt$.

a) $2x\sqrt{\cos x^2}$

b) $\dfrac{x \cos x^2}{\sqrt{\sin x^2}}$

c) $2\sqrt{\cos x^2}$

d) $2x\sqrt{\sin x^2}$

e) $\dfrac{\cos x^2}{2\sqrt{\sin x^2}}$

15. Evaluate $\displaystyle\int 4 \tan^3 x \cdot \sec^2 x\ \ dx$.

a) $4 \tan^4 x + C$

b) $\dfrac{1}{4} \tan^4 x + C$

c) $\tan^4 x + C$

d) $\tan^4 x \cdot \sec^2 x + C$

e) $\tan^4 x \cdot \sec^3 x + C$

16. Evaluate $\displaystyle\int_0^1 x\sqrt{x^2 + 1}\ \ dx$.

a) $\dfrac{2\sqrt{2}}{3}$

b) $\dfrac{2\sqrt{2} - 1}{2}$

c) $\dfrac{2\sqrt{2} - 2}{3}$

d) $\dfrac{1}{3}$

e) $\dfrac{2\sqrt{2} - 1}{3}$

SHORT-ANSWER PROBLEMS, 17–20. Each problem is worth 5 points.
Evaluate each of the following limits. If the limit is infinite, indicate $+\infty$ or $-\infty$.
Circle your answer.

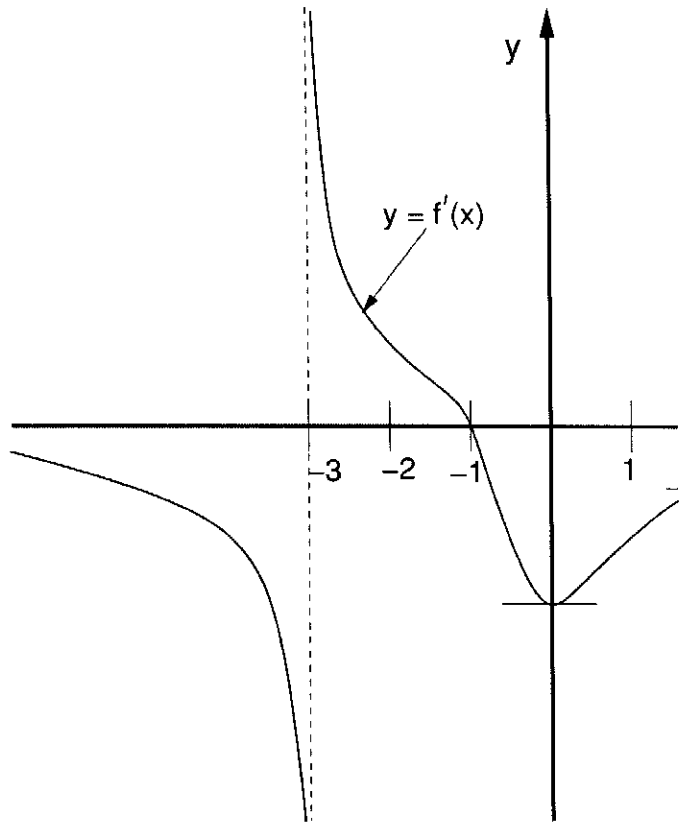
17. $\displaystyle\lim_{x\to 2^+} \frac{x^2 - 4}{x^2 - 5x + 6}$

18. $\displaystyle\lim_{x\to -3^-} \frac{x + 4}{(x + 3)^2}$

19. $\lim_{x \to 2} \dfrac{\sqrt{2x+5} - 3}{x - 2}$

20. $\lim_{\theta \to 0} \dfrac{\sin 4\theta}{\theta \cos 5\theta}$

21. (11 points) If $y = f(x)$ is a **continuous** function for all real numbers, and the graph of its **derivative function** $f'(x)$ is presented below, answer the following questions about the behavior of the graph of $f(x)$



Note again: These questions refer to the function $y = f(x)$ as obtained from the graph of $y = f'(x)$ shown above.

a) List all intervals where the function f is increasing._____

b) List all intervals where the function f is decreasing._____

c) List the x-value(s) for any/all local maxima._____

d) List the x-value(s) for any/all local minima._____

e) List all intervals where the function f is concave up._____

f) List all intervals where the function f is concave down._____

g) List the x-values for any/all inflection points._____

22. (10 points) Given two functions, $y = x^3$ and $y = 4x$.

a) Make a sketch that represents their graphs on the same axes. Label each point of intersection.

b) Set up the integral that represents the area enclosed by these two curves.

c) Evaluate the integral to find the area. Show all work.

23. (13 points)

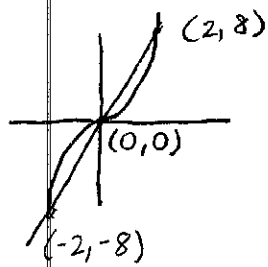
a) Draw a picture of the region bounded by the curves $y = 4 - 2x$ and $y = 4 - x^2$. Label each point of intersection.

b) Set up **BUT DO NOT EVALUATE** the integral that represents the volume of the solid of revolution obtained by rotating this region about the x-**axis**.

c) Set up **BUT DO NOT EVALUATE** the integral that represents the volume of the solid of revolution obtained by rotating this region about the y-**axis**.

FINAL EXAM- FORM A

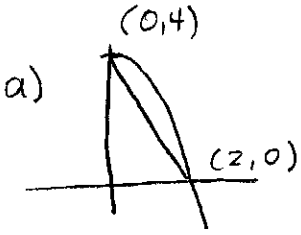
1. E
2. E
3. A
4. B
5. D
6. E
7. B
8. A
9. C
10. B
11. D
12. C
13. C
14. D
15. C
16. E
17. -4
18. $+\infty$
19. $1/3$
20. 4
21. a $(-3,-1)$
    b. $(-\infty,-3),(-1,\infty)$
    c. $x=-1$
    d. $x=-3$
    e. $(0,2)$
    f. $(-\infty,-3),(-3,0),(2,\infty)$
    g. $x=0, x=2$
22. a)



b) $A = 2\int_0^2 (4x-x^3)dx$

c) 8

23. a)



b) $V = \pi\int_0^2 [(4-x^2)^2-(4-2x)^2]dx$

or

$V = 2\pi\int_0^4 y\left[\sqrt{4-y}-(2-\frac{y}{2})\right]dy$

c) $V = 2\pi\int_0^2 x[(4-x^2)-(4-2x)]dx$

or

$V = \pi\int_0^4 [(\sqrt{4-y})^2-(2-\frac{y}{2})^2]dy$