MATH 110 (HUFF) FINAL EXAM DEC. 16, 2003

Name____________________________ ID #__________________ Section #______

The examination consists of **25** multiple choice questions. For each problem, please fill in the bubble on the scantron sheet and circle the correct answer on your examination. Each problem is worth **6** points.

THE USE OF CALCULATORS IS <u>NOT</u> PERMITTED IN THIS EXAMINATION.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE **25** PROBLEMS ON **14** PAGES (INCLUDING THIS ONE).

1. Find $\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 2x}$.

   a) 0

   b) $\frac{5}{3}$

   c) $\frac{7}{2}$

   d) The limit does not exist.

2. Find all asymptotes of the graph of $f(x) = \frac{(x-1)(3x+3)}{x^2 + 2x}$.

   a) Vertical asymptotes $x = 0$ and $x = -2$, horizontal asymptote $y = 3$.

   b) Vertical asymptotes $x = 1$ and $x = -\frac{3}{2}$, horizontal asymptote $y = 3$.

   c) Vertical asymptote $x = 0$, horizontal asymptotes $y = 1$ and $y = \frac{3}{2}$.

   d) No vertical asymptotes, horizontal asymptote $y = 1$.

3. The function

$$f(x) = \frac{|x|}{x+1}$$

is continuous except at the point(s):

a) $x = 0$

b) $x = 0$ and $x = -1$

c) $x = -1$

d) f is continuous everywhere.

4. Find the equation of the line tangent to the graph of

$$y = 3 + e^{x^2+2x}$$

at the point $(0, 4)$.

a) $y = x + 4$

b) $y = 2x - 5$

c) $y = 4 - x$

d) $y = 2x + 4$

5. Find $\frac{dy}{dx}$ when $x = 2$ and $y = 1$ if

$$xy^2 = x + y^3 - 1.$$

a) $\frac{dy}{dx} = 0$

b) $\frac{dy}{dx} = -1$

c) $\frac{dy}{dx} = 2$

d) $\frac{dy}{dx} = \frac{1}{2}$

6. What is $f'(x)$ if $f(x) = \ln(e^{2x} + 5)$?

a) $f'(x) = \ln(2e^{2x})$

b) $f'(x) = \frac{2e^{2x}}{e^{2x} + 5}$

c) $f'(x) = \frac{1}{e^{2x} + 5}$

d) $f'(x) = \frac{2}{e^{2x}} + \frac{1}{5}$

7. What is the *second* derivative of $f(x) = \ln\sqrt{1+x}$?

a) $f''(x) = \dfrac{1}{\sqrt{1+x}}$.

b) $f''(x) = \dfrac{-1}{2(1+x)^2}$.

c) $f''(x) = \dfrac{1}{2}\ln(\ln(1-x))$.

d) $f''(x) = \dfrac{-1}{2(1+x)^{3/2}}$.

8. Suppose x and y are differentiable functions of t, and are related by the equation

$$y^3 = xy + 6.$$

Find $\dfrac{dy}{dt}$ when $x = 1$, $y = 2$, and $\dfrac{dx}{dt} = 3$.

a) $\dfrac{dy}{dt} = 0$

b) $\dfrac{dy}{dt} = \dfrac{6}{11}$

c) $\dfrac{dy}{dt} = \dfrac{1}{2}$

d) $\dfrac{dy}{dt} = -\dfrac{3}{10}$

9. Let $f(x) = 2x^3 - 3x^2 + 2x + 1$. Find the point(s) on the graph of f at which the slope of the tangent line is equal to 2.

    a) $(0, 1)$ and $(1, 2)$
    b) $(0, 1)$ and $(2, -1)$
    c) $(1, -2)$ and $(2, 9)$
    d) There are no such points

10. Find the interval(s) where the function

$$f(x) = \frac{\ln x}{x} \quad (x > 0)$$

is increasing.

    a) f is always increasing.
    b) f is increasing on $(0, 1)$ and on (e, ∞).
    c) f is increasing on $(0, e)$.
    d) f is never increasing.

11. If the *second* derivative of $f(x)$ is

$$f''(x) = \frac{x^2 - x}{x - 3},$$

where is the graph of f concave upward?

a) concave upward on $(-3, \infty)$.

b) concave downward everywhere.

c) concave upward on $(0, 1)$ and on $(3, \infty)$.

d) concave upward everywhere.

12. Find the absolute maximum value of $f(x) = xe^{-x}$.

a) 0

b) $\frac{1}{e}$

c) $\frac{2}{e^2}$

d) There is no absolute maximum.

13. If a bank pays interest at rate $r = 6\%$ compounded monthly, what is the effective interest rate r_{eff} ?

a) $r_{\text{eff}} = 1 + (0.005)^{12}$

b) $r_{\text{eff}} = e^{0.06} - 1$

c) $r_{\text{eff}} = (1.06)^{12} - 1$

d) $r_{\text{eff}} = (1.005)^{12} - 1$

14. A radioactive isotope has half life 15 years. How long will it take a sample to decay to one-eight of the original amount?

a) 15^3 years.

b) 45 years.

c) $\dfrac{120}{\ln 2}$ years.

d) $15 \ln 2$ years.

15. What is $\int \left(e^{2x} - \frac{3}{x} + \frac{1}{x^3} \right) dx$?

a) $\frac{1}{2}e^{2x} - 3\ln|x| - \frac{1}{2x^2} + C$

b) $2e^{2x} + \frac{3}{x^2} + \frac{3}{x^4} + C$

c) $e^{x^2} - \frac{6}{x^2} + \frac{4}{x^4} + C$

d) $\frac{1}{2}e^{2x} - 3\ln|x| + \frac{1}{3x^2} + C$

16. If $f(1) = 2$ and $f'(x) = \frac{\ln x}{x}$, what is $f(x)$?

a) $f(x) = \ln(\ln x) + 2 - \ln 2.$

b) $f(x) = 2\ln x + 2.$

c) $f(x) = \frac{1 - \ln x}{x^2} + 1.$

d) $f(x) = \frac{1}{2}(\ln x)^2 + 2.$

17. Find $\int_0^1 xe^{2x^2}\,dx$.

a) $\frac{1}{4}(e^2 - 1)$

b) $4e^2$

c) $\frac{1}{2}(2e - 1)$

d) $\frac{1}{4}(e^2 + e + 1)$

18. What is $\int_0^4 x\sqrt{x^2+9}\,dx$?

a) 24

b) $\frac{65}{2}$

c) 32

d) $\frac{98}{3}$

19. Find $\dfrac{dy}{dx}$ if $y = (x+1)^{2x} \quad (x > -1)$. (**Hint:** Use logarithmic differentiation.)

a) $\dfrac{dy}{dx} = 2x(x+1)^{2x-1}$

b) $\dfrac{dy}{dx} = (x+1)^{2x}\left[2\ln(x+1) + \dfrac{2x}{x+1}\right]$

c) $\dfrac{dy}{dx} = (2\ln(x+1))(x+1)^{2x}$

d) $\dfrac{dy}{dx} = \dfrac{1}{2x+1}(x+1)^{2x+1}$

20. Assume the percentage of students owning cell phones t years from now is given by the logistic function

$$f(t) = \frac{100}{1+3e^{-0.1t}} \, .$$

For what value of t will 50% of students own cell phones?

a) $t = (10\ln 3)$ years

b) $t = \dfrac{10}{\ln 3}$ years

c) $t = 5e^{0.3}$ years

d) $t = \dfrac{10\ \ln 2}{\ln 3}$ years

21. What is the average value of $f(x) = \frac{1}{x}$ over the interval $[1, 5]$?

a) $\frac{3}{5}$

b) $\frac{\ln 5}{4}$

c) $\frac{\ln 4}{2}$

d) $\ln 5$

22. With appropriate units, the supply function for a certain product is

$$p = S(x) = \frac{3}{100}x^2 + \frac{2}{10}x + 7,$$

where p is the price and x is the quantity. Find the producers' surplus if $\overline{x} = 10$ and $\overline{p} = 12$.

a) 40

b) 35

c) 30

d) 25

23. What is the area that is completely enclosed by the graph of $y = x^2$ and the horizontal line $y = 1$?

a) $\frac{27}{16}$

b) $\frac{4}{3}$

c) $\frac{3}{2}$

d) 2

24. If an continuous income stream of \$1000 per year is invested at 10% per year compounded continuously, then the value of the account after 5 years will be

$$A = 1000e^{0.5} \int_0^5 e^{-0.10t}\, dt.$$

Compute A.

a) $1,000e$ dollars

b) $10,000e^{0,5}$ dollars

c) $1,000(1 + e^{0.5})$ dollars

d) $10,000(e^{0.5} - 1)$ dollars

25. For $x > 0$, let

$$G(x) = \int_0^x e^{t^2}\, dt.$$

That is, for each $x > 0$, $G(x)$ is the area under the graph of $y = e^{t^2}$, above the t-axis, and between the lines $t = 0$ and $t = x$. What is $G'(x)$?

a) $G'(x) = e^{x^2}$

b) $G'(x) = 2xe^{x^2}$

c) $G'(x) = e^{2x}$

d) $G'(x) = e^{2x-1}$

26. KEY: 1-c, 2-a, 3-c, 4-d, 5-a, 6-b, 7-b, 8-b, 9-a, 10-c, 11-c, 12-b, 13-d, 14-b, 15-a, 16-d, 17-a, 18-d, 19-b, 20-a, 21-b, 22-c, 23-b, 24-d, 25-a.