1.1 Basic Equations

- Fractional Equations:
  - Find numbers that would make the denominator equal to 0
  - Multiply by LCD
- Using Radicals: Solution of $x^n = a$ is
  - $x = \pm\sqrt[n]{a}$ if n is even
  - $x = \sqrt[n]{a}$ if n is odd

1.2 Modeling with Equations

- Read the problem several times
- Figure out what you have to find
- *Define variables*
- Write down all the information, sketch a diagram if necessary

1.3 Quadratic Equations

- Factoring
- Completing the square
- Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- Discriminant: $D = b^2 - 4ac$
  - if $D > 0$ then the equation has two real solutions
  - if $D = 0$ then the equation has one real solution (of multiplicity 2)
  - if $D < 0$ then the equation has two complex solutions

1.4 Complex Numbers

- Addition/Subtraction: $(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i$
- Multiplication: $(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$
- Conjugate of $a + bi$: $a - bi$
- Division: Multiply by conjugate: $\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \cdot \frac{c-di}{c-di} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$
- Square roots of negative numbers: $\sqrt{-a} = \sqrt{a} \cdot i$

1.5 Other Types of Equations

- Polynomial Equations
- Factoring by grouping
- Equations with Radicals: Isolate the root on one side, then square/cube/etc.
- Substitution

1.6 Inequalities

- Make the right side equal to 0
- Find the critical points: where the left side is either 0 or undefined
- Put the critical points on the number line
- Use test points
- If multiplying/dividing by a negative number reverse the direction of the inequality

1.7 Absolute Value Equations and Inequalities

- Equations:
  - Isolate the absolute value on one side
  - $|x| = C$ implies $x = \pm C$
- Inequalities:
  - Isolate the absolute value on one side
  - $|x| < C$ implies $-C < x < C$
  - $|x| > C$ implies $x > C$ or $x < -C$

2.1 The Coordinate Plane

- Distance formula: $d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Midpoint formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

2.2 Graphs of Equations in two Variables

- Finding intercepts:
  - x-intercepts: set y equal to 0
  - y-intercepts: set x equal to 0
- Equation of a circle: $(x - x_C)^2 + (y - y_C)^2 = r^2$
- Symmetry:
  - x-axis: equation unchanged if y is replaced by $-y$
  - y-axis: equation unchanged if x is replaced by $-x$
  - origin: equation unchanged if x is replaces by $-x$ and y by $-y$

2.4 Lines

- Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Slope of a vertical line is undefined, slope of a horizontal line is 0
- Point-Slope Form: $y - y_P = m(x - x_P)$
- Slope-Intercept Form: $y = mx + b$
- Vertical line: $x = a$, Horizontal line: $y = b$
- Parallel lines: same slope
- Perpendicula lines: $m_2 = -\frac{1}{m_1}$

2.5 Modeling: Variation

- Direct variation: $y = kx$
- Inverse variation: $y = \frac{k}{x}$
- Joint variation: Multiple instance of the two kinds above

3.1 What is a function

- Domain: all possible x-values. Finding the domain: Find potential problems that could make the function undefinend such as
  - Roots (even power): radicand cannot be negative
  - Fractions: denominator cannot be 0
- Range: all possible y-values: Finding the range: Graph the function

3.2 Graphs of Functions

- Piece-wise defined functions
- Vertical line test: if a vertical line crosses two points of the graph, then the graph is not a function
- Test equations if they define a function:
  - Solve for y
  - Check if y is unique
- Memorize the graphs of the functions on page 232

3.3 Increasing and Decreasing Functions, Average Rate of Change

- Increasing and Decreasing Functions
- Average rate of change: $= \frac{f(b)-f(a)}{b-a}$

3.4 Transformations of Functions

- Vertical shift: $f(x) + c$
- Horizontal shift: $f(x - c)$
- Reflection in x-axis: $-f(x)$
- Reflection in y-axis: $f(-x)$
- Vertical stretch($c > 1$)/shrink($c < 1$) by factor c: $cf(x)$
- Horizontal stretch($c < 1$)/shrink($c > 1$) by factor $1/c$: $f(cx)$
- Even function: symmetric with respect to y-axis: $f(x) = f(-x)$
- Odd function: symmetric with respect to origin: $-f(x) = f(-x)$

3.5 Quadratic Functions, Maxima and Minima

- $f(x) = ax^2 + bx + c$
- Maximum if $a < 0$, Minimum if $a > 0$
- x-value of Max/Min is $-\frac{b}{2a}$

Focus on Modeling: Modeling with functions, pages 295-307

- Draw a picture if possible
- Identify all variables
- Determine a model
- Use the model
- Make sure you have answered the question asked

3.6 Combining Functions

- Addition/Subtraction/Multiplication/Division of functions: Add/Subtract/etc. the y-values
- Composition of functions: $(f \circ g)(x) = f(g(x))$

3.7 One-to-one Functions and their Inverses

- One-to-one functions: no two differenct x-values have the same y-value
- Horizontal line test: if a horizontal line crosses two points of the graph, the function is not one-to-one
- Inverse function: $f^{-1}(y) = x \leftrightarrow f(x) = y$
- Finding inverses:
  - Reverse x and y
  - Solve for y
- Properties:
  - $f(f^{-1}(x)) = x$
  - $f^{-1}(f(x)) = x$

4.1 Polynomial Functions and their Graphs

- End behavior:
  - Look at leading coefficient/exponent and check sign
  - If polynomial is factored, check sign of each factor and multiply
- Graphing a polynomial:
  - Factor
  - Find x- and y-intercepts
  - Find end behavior
  - Either use test points between the intercepts or memorize the shape around zeros depending on the multiplicity:
    - If multiplicity is 1, then it crosses the x-axis in a straight line
    - If multiplicity is even, then it turns back around
    - If multiplicity is odd > 1, then it "squiggles" through the x-axis

4.2 Dividing Polynomials

- Long Division: Make sure to fill in missing powers. Write answers in the form $Quotient + \frac{remainder}{divisor}$
- Synthetic Division: Only works for division by $(x - c)$. Again make sure to fill in 0's for missing powers Will not be covered, but you may use it if you remember it.
- Factor Theorem: c is a zero of $P \leftrightarrow (x - c)$ is a factor of $P(x)$

4.3 No longer covered in Math 22

4.4 No longer covered in Math 22

4.5 Rational Functions

- Horizontal asymptotes: n is the degree of the numerator, m is the degree of the denominator
  - $n > m$: no horizontal asymptote
  - $n = m$: horizontal asymptote is $y = \frac{a_n}{b_m}$
  - $n < m$: horizontal asymptote is $y = 0$
- Vertical asymptotes: zeros of the denominator (that do not cancel with the numerator)
- Graphing rational functions:
  - Factor numerator and denominator
  - Find x- and y-intercepts
  - Find horizontal and vertical asymptotes
  - Either use test points between intercepts/vertical asymptotes or use the shape around vertical asymptotes/intercepts to determine the shape of the graph
- Slant asymptote: only exists if the degree of the numerator is one greater than the degree of the denominator: use long/synthetic division

5.1 Exponential Functions

- $f(x) = a^x$, memorize the graph:
  - Horizontal asymptote $y = 0$
  - no vertical asymptote
  - Domain $= (-\infty, \infty)$
  - Range $= (0, \infty)$
- Compound interest formula: $A(t) + P(1 + \frac{r}{n})^{nt}$
- Continuously compounded interest: $A(t) = e^{rt}$

5.2 Logarithmic Functions

- Definition of logarithm: $\log_b a = x \leftrightarrow b^x = a$
- Properties:
  - $\log_b 1 = 0$
  - $\log_b b = 0$
  - $\log_b b^x = x$
  - $b^{\log_b x} = x$
- $f(x) = \log_b x$, memorize the graph:
  - Vertical asymptote: $x = 0$
  - no horizontal asymptote
  - Domain $= (0, \infty)$
  - Range $= (-\infty, \infty)$
- Finding the domain of logarithmic function: logarithms only defined for positive numbers
- Common log: $\log x = \log_1 0x$
- Natural log: $\ln x = \log_e x$

5.3 Laws of Logarithms

- $\log_b(xy) = \log_b x + \log_b y$
- $\log_b \frac{x}{y} = \log_b x - \log_b y$
- $\log_b x^y = y \log_b x$
- no laws for $\log_b(x + y)$ or $\log_b x \cdot \log_b y$
- Change of base: $\log_b x = \frac{\log_c x}{\log_c b}$ where c can be any positive number

5.4 Exponential and Logarithmic Equations

- Solving exponential equations:
  - Isolate the exponential term on one side
  - Take logarithm of both sides:
    - If there is only one exponential term, use that base for the log
    - If there is an exponential term on both sides, use either the common or natural log
  - Pull the exponent to the front and solve the equation
- Solving logarithmic equations:
  - If there are multiple logarithmic terms, combine them into one using logarithmic laws
  - Isolate the logarithmic term on one side
  - Raise the base of the logarithm to the left and the right side of the equation
  - Use the property $b^{\log_b x} = x$ to get rid of the log
  - Solve the equation
- Two special cases of exponential equations:
  - Combination of exponential and polynomial terms: try to factor
  - Sum of multiply exponential terms: try to use substitution

5.5 Modeling with Exponential and Logarithmic Functions

- Exponential growth model: $n(t) = n_0 e^{rt}$
- To solve any problem you usually have to find n_0 and r
- Formulas and logarithmic scales

9.1 Sequences and Summation Notation

- A few commonly used terms:
  - $(-1)^n$ or $(-1)^{n+1}$ for sequences alternating in sign
  - $2n$ for even numbers
  - $2n-1$ for odd numbers
- Recursively defined sequences, Fibonacci Numbers
- Partial sums
- Sigma notation: $\sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$
- Properties:
  - $\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$
  - $\sum_{k=1}^{n}(a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$
  - $\sum_{k=1}^{n} ca_k = c\sum_{k=1}^{n} a_k$

9.2 Arithmetic Sequences

- Arithmetic sequence: a, $a+d$, $a+2d$, $\ldots$
- $a_n = a + d(n-1)$
- Gauss: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
- Partial sums of an arithmetic sequence: $S_n = \frac{n}{2}(2a + d(n-1))$

9.3 Geometric Sequences

- Geometric sequence: a, ar, ar^2, $\ldots$
- $a_n = ar^{n-1}$
- Partial sums of an arithmetic sequence: $S_n = a\frac{1-r^n}{1-r}$
- Sum of an infinite geometric series: $\sum_{k=1}^{n} ar^{k-1} = \frac{a}{1-r}$ if $|r| < 1$