

MATH 140

NAME \_\_\_\_\_

EXAM I

STUDENT NUMBER \_\_\_\_\_

OCTOBER 8, 2003

INSTRUCTOR \_\_\_\_\_

FORM A

SECTION NUMBER \_\_\_\_\_

This examination will be machine processed by the University Testing Service. Use only a number 2 pencil on your scantron. On your scantron identify your name, this course (Math 140) and the date. Code and blacken the corresponding circles on your scantron for your student I.D. number, section number and your **test form**.

There are 10 multiple choice questions worth a total of 50 points. For each problem **five** possible answers are given, only one of which is correct. **Circle** the correct answer in your exam booklet **and blacken** the corresponding space on the **scantron form**. Mark only one choice; darken the circle completely (you should not be able to see the letter after you have darkened the circle). Check frequently to be sure the problem number on the test is the same as the problem number of the scantron. There are **4** partial credit questions worth a total of 50 points. **In order to obtain full credit for these problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is shown to the right of the question number.

**ALL CALCULATORS, NOTES, BOOKS, ETC. ARE FORBIDDEN.**

MC (50 Pts.) _____
11. (10 Pts.) _____
12. (10 Pts.) _____
13. (15 Pts.) _____
14. (15 Pts.) _____
<b>Total</b> _____

**Do not  
write in  
the box to  
the left.**

5 pts 1. Calculate the limit:

$$\lim_{x \rightarrow 7} \frac{x^2 - 8x + 7}{x^2 - 9x + 14}$$

- a) 0
- b)  $\infty$
- c)  $-\infty$
- d)  $6/5$
- e)  $1/2$

5 pts 2. Calculate the one-sided limit:

$$\lim_{x \rightarrow 2^-} \frac{|x^2 - 4|}{2 - x}$$

- a) 0
- b) 4
- c)  $-4$
- d) 8
- e) the limit does not exist

5 pts 3. Calculate the limit:

$$\lim_{t \rightarrow 0} \frac{\sqrt{9+t} - 3}{t}$$

- a) 0
- b) 1/6
- c) -1/8
- d) 1/3
- e) the limit does not exist

5 pts 4. Differentiate  $3\sqrt{x^2 + x} + 2$  with respect to  $x$ .

- a)  $\frac{6x + 3}{2\sqrt{x^2 + x}}$
- b)  $\frac{3}{2\sqrt{2x + 1}}$
- c)  $\frac{1}{2}(x^2 + 1)^{-1/2}$
- d)  $3 + \frac{3}{2}x^{-1/2}$
- e)  $\frac{3}{2\sqrt{2x + 1}} + 2$

5 pts 5. Suppose  $f(x)$  is continuous on the interval  $[2, 4]$  with  $f(2) = 7$  and  $f(4) = 1$ . Which one of the following statements **must be true**?

a)  $f(x)$  is differentiable on  $(2, 4)$

b)  $f(3) = 4$

c)  $1 \leq f(x) \leq 7$  for all  $x$  in  $[2, 4]$

d)  $f(x) = 2$  for some  $x$  in  $(2, 4)$

e)  $f(x) = 0$  for some  $x$  in  $(2, 4)$

5 pts 6. If  $s(x) = \sin^2(\cos x)$  then  $\frac{ds}{dx}$  is,

a)  $2 \sin(\cos x) - \sin^2(\sin x)$

b)  $-\cos^2(\cos x) \cdot \sin x$

c)  $\sin(x) \cdot \cos^2 x$

d)  $-2 \sin(\cos x) \cdot \sin x$

e)  $-2 \sin(\cos x) \cdot \cos(\cos x) \cdot \sin x$

- 5 pts 7. Find an equation of the line tangent to the graph of the function  $y = \frac{x^2}{x+2}$  at the point  $(2, 1)$ .

a)  $y = 2x^2 - 4x + 1$

b)  $y = \frac{3}{4}x + \frac{5}{4}$

c)  $y = \frac{3}{4}x - \frac{1}{2}$

d)  $y = 4x - 2$

e)  $y = 4x - 7$

- 5 pts 8. Which of the following is true of

$$f(x) = \frac{x^2 - 9}{x^2 - 5x + 6}?$$

- a) It has a vertical asymptote at  $x = 2$ .  
b) It has a removable discontinuity at  $x = 2$ .  
c) It has a jump discontinuity at  $x = 2$ .  
d) It is continuous at  $x = 2$ .  
e) None of the above are true.

5 pts 9. If  $g(x) = 3x + 2$ , what is the biggest value of  $\delta$  that guarantees that  $|g(x) - 5| < 0.06$  whenever  $0 < |x - 1| < \delta$ ?

- a) 0.01
- b) 0.02
- c) 0.03
- d) 0.06
- e) None of the above

5 pts 10. Let  $f(x) = \begin{cases} (x - 1)^2 & \text{if } x \geq 1 \\ cx & \text{if } x < 1 \end{cases}$ . Find the value of  $c$  that makes  $f$  continuous at  $x = 1$ .

- a) 0
- b) 1
- c) 2
- d) 3
- e) The function is discontinuous at  $x = 1$  for all values of  $c$ .

- 10 pts 11. Find the limit. You are not allowed to use L'Hopital's Rule. Mathematically correct notation is required for full credit.

$$\lim_{x \rightarrow 0} \frac{3x + \sin(5x)}{2x}$$

10 pts 12. Part (a) State the limit definition of the derivative of a function  $f(x)$ .

4 pts

6 pts

Part (b) Use the definition of derivative to compute  $f'(x)$  if  $f(x) = \frac{1}{x}$ . (Warning: Just giving the answer will earn you zero points! You **must use the definition from part (a)**.)

15 pts 13. A particle moves along a line and  $s(t) = t^3 - 3t^2 + 4$  gives the position (in feet) of the particle at time  $t$  (in seconds).

3 pts (a) Find the velocity  $v(t)$  of the particle.

3 pts (b) What is the velocity after 3 seconds?

3 pts (c) When is the particle at rest?

3 pts (d) When is the particle moving in the positive direction?

3 pts (e) Find the total distance traveled in the first 3 seconds (i.e., from  $t = 0$  to  $t = 3$ ).

15 pts 14. Draw a graph of a function  $f(x)$  that satisfies the following properties. The graph should be large enough for all the required features to be easily recognizable. You do not have to provide any formulas.

1) The domain of  $f(x)$  is  $[-5, 5]$  and  $f(x)$  is continuous at all points of the domain except at  $x = 0$  and  $x = 2$ .

2)  $\lim_{x \rightarrow -5^+} f(x) = -2$  and  $\lim_{x \rightarrow 5^-} f(x) = 1$ .

3) The function  $f(x)$  has a removable discontinuity at  $x = 0$  and  $\lim_{x \rightarrow 0} f(x) = 2$ .

4) The function  $f(x)$  has a jump discontinuity at  $x = 2$  with  $\lim_{x \rightarrow 2^-} f(x) = 1$  and  $\lim_{x \rightarrow 2^+} f(x) = 3$ .

5) The function  $f(x)$  is **not differentiable** at  $x = 3$ .

6)  $f'(x) > 0$  for  $-5 < x < 0$ .