

Answer Keys to Final Exam, Math 251, Spring 2012

Form A:

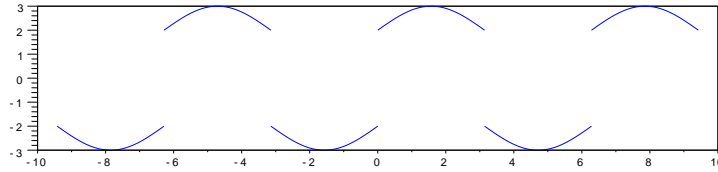
Problems	1	2	3	4	5	6	7	8	9
Answers	c	c	b	a	d	a	b	b	c

Problem 10: (a) True (b) False (c) False (d) True (e) False

Problem 11: (a) 4 (b) 1 (c) 6 (d) 3 (e) 6

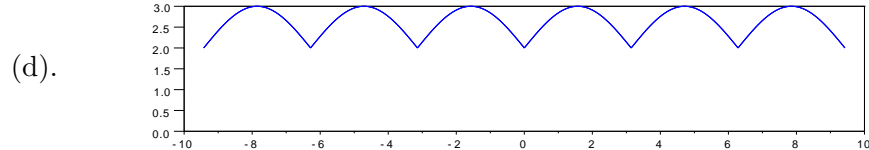
Problem 12: (a). $\lambda_n = \left(\frac{n\pi}{5}\right)^2$, $X_n(x) = \cos \frac{n\pi x}{5}$, $n = 1, 2, 3, \dots$ (6 pts + 6 pts)
 (b). Yes. $X_0(x) = 1$.

Problem 13: (a).



(b). (ii)

(c). At $x = -\pi$, it converges to 0. At $x = \frac{3}{2}\pi$, it converges to -3.



(e). 2.

Problem 14: (a). True

(b). $u(x, t) = \sum_{n=1}^{+\infty} C_n \cos \frac{10n\pi t}{L} \sin \frac{n\pi x}{L}$

(c). $C_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$

Problem 15: (a). $v(x) = 5x + 32$

(b). General form $u(x, t) = 5x + 32 + \sum_{n=1}^{+\infty} C_n e^{-\frac{3}{16}n^2\pi^2 t} \sin \frac{n\pi x}{4}$. (3 pts)

Solution: $u(x, t) = 5x + 32 - 40e^{-\frac{3}{4}\pi^2 t} \sin \frac{\pi x}{2} - 24e^{-12\pi^2 t} \sin(2\pi x)$. (7 pts)

(c). 42.

(d). The limit is the same, because the steady-state does not change.

Form B:

Problems	1	2	3	4	5	6	7	8	9
Answers	c	b	a	c	a	d	d	c	a

Problem 10: (a) False (b) True (c) True (d) False (e) True

Problem 11: (a) 5 (b) 3 (c) 2 (d) 1 (e) 2

Problem 12-15: same as Form A, except Problem 13(b): (iv).