MATH 251
Final Exam
May 7, 2008

ANSWER KEY

1. A

2. C

3. A

4. D

5. B

6. C

7. B

8. (a) $y(t) = t^2 - 1 + 5t^{-2}$
(b) $(-\infty, 0)$

9. $y(t) = e^{-2t}\cos(2t) + e^{-2t}\sin(2t) + \frac{1}{2}u_{2\pi}(t)\,e^{-2(t-2\pi)}\sin(2t)$

10. (a) $4X'' + \lambda X = 0$, and $T'' + (\lambda + 5)T = 0$
(b) $X'(0) = 0,\ X(20) = 0$
(c) No

11. Eigenvalues are $\lambda = 0$ and $\lambda = n^2$, $n = 1, 2, 3, \ldots$;
Eigenfunctions are $X_0(x) = 1$, and $X_n(x) = \cos(nx)$, $n = 1, 2, 3, \ldots$

12. (b) It is a sine series.

(c) $a_m = 0,\ m = 0, 1, 2, \ldots;\ b_n = \int_0^2 x^2 \sin\left(\frac{n\pi x}{2}\right)dx,\ n = 1, 2, 3, \ldots$

(d) $f(0) = 0,\ f(2) = 0,\ f(3) = -1$

13. (a) $u(x,t) = 2e^{-4\pi^3 t/9}\sin\left(\frac{\pi x}{3}\right) + 4e^{-4\pi^2 t}\sin(\pi x) - 10e^{-9\pi^2 t}\sin\left(\frac{3\pi x}{2}\right)$

(b) $\lim_{t\to\infty} u(x,t) = 0$