

ANSWER KEY

1. C

2. A

3. B

4. D

5. a) Node, asymptotically stable (a “nodal sink”)
b) Saddle point, unstable
c) Spiral point, unstable (a “spiral source”)
d) Center, stable (but not asymptotically stable)
e) Node, unstable (a “nodal source”)

6. a) $a(x)$ and $e(x)$
b) $b(x)$ and $c(x)$

7.
$$y(t) = \frac{1}{t} \left(\frac{1}{2} e^{2t} + 4 - \frac{1}{2} e^2 \right)$$

8. a) Eigenvalues are $\lambda = \frac{(2n-1)^2 \pi^2}{4}$, $n = 1, 2, 3, \dots$

b) Eigenfunctions are $X_n(x) = \cos\left(\frac{(2n-1)\pi}{2}x\right)$, $n = 1, 2, 3, \dots$

9. a) $(1,1), (1,-1), (-1,1), (-1,-1)$

b)
$$x' = \begin{bmatrix} -2 & -2 \\ -2 & 2 \end{bmatrix} x$$

- c) Saddle point, unstable

10.
$$\begin{cases} \lambda X'' + 5X' - 7X = 0, & X'(0) = 0, & X(L) = 0 \\ \lambda T' + tT = 0 \end{cases}$$

11.
$$v(x) = 10x - \frac{5}{2}$$

12. a) $v(x) = 10x + 20$

b) $u(x, t) = 10x + 20 - e^{-3\pi^3 t/2} \sin\left(\frac{\pi x}{2}\right) + 15e^{-24\pi^2 t} \sin(2\pi x)$

c) $\lim_{t \rightarrow \infty} u(1, t) = 30$

13. b) $b_n = \frac{-4}{(2n-1)\pi}, n = 1, 2, 3, \dots$

c) $f(-1) = 1, f(0) = 0, f(3) = 1$