Math 251 Spring 2004 Final Exam

ANSWERS

1. a) True; b) True; c) False; d) True; e) False; f) True

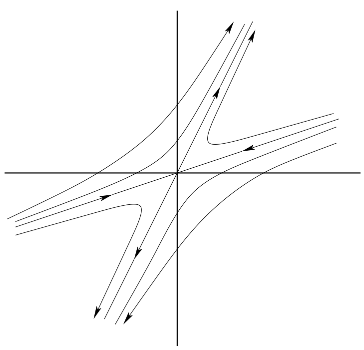
2. a) $\mu(t) = t^{-3}$; b) $c = 3$; c) $Y(t) = Ate^{-2t} + Bt^2 + Ct + D$.

3. a) $T = \dfrac{2\pi}{3}$ (sec); b) $\mu = \sqrt{5}$ (rad/sec); c) It is critically damped.

4. $y(t) = u_1(t)(t-1)e^{3(t-1)} - 2u_3(t)(t-3)e^{3(t-3)}$

5. $\begin{cases} x_1' &= x_2 \\ x_2' &= \frac{-5}{2}x_1 + x_2 \end{cases}$ or $\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ \frac{-5}{2} & 1 \end{bmatrix} \mathbf{x}$

6. a) $\mathbf{x}(t) = C_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{4t} + C_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{-t}$
b) It is an unstable saddle point.
c) $\alpha = -1$
d)



7. a) $\mathbf{x}(t) = \begin{bmatrix} \cos t - 4\sin t \\ 4\cos t + \sin t \end{bmatrix} e^{3t}$
b) It is an unstable spiral point.

8. a) The critical points are $(0,-2), (1,1), (-2,-2)$.
b) $(0,-2)$ is an unstable saddle point, $(0,0)$ is an unstable spiral point, and $(-2,-2)$ is an asymptotically stable node.

9. $\begin{cases} \lambda X'' - X' + 5x^3 X = 0 \\ \lambda T' - T = 0 \end{cases}$

10. The eigenvalues are $\lambda = \dfrac{1}{4}. \dfrac{4}{4}, \dfrac{9}{4}, ..., \dfrac{n^2}{4}, ...$ Their eigenfunctions are $y_n(x) = C_n \cos \dfrac{nx}{2}$, $n = 1, 2, 3, 4, ...$
b) Yes, $\lambda = 0$ is an eigenvalue. Any nonzero constant function is a corresponding eigenfunction.

11. For $m = 0, 1, 2, 3, ...,$ $a_m = \frac{1}{2}\left[\int_{-2}^{-1} (-2-x)\cos\frac{m\pi x}{2}\,dx + \int_{-1}^{1} x\cos\frac{m\pi x}{2}\,dx + \int_{1}^{2} (2-x)\cos\frac{m\pi x}{2}\,dx\right]$

or Just state that $a_m = 0$ for all m (by noting that $f(x)$ is an odd function).

For $n = 1, 2, 3, ...,$ $b_n = \frac{1}{2}\left[\int_{-2}^{-1} (-2-x)\sin\frac{n\pi x}{2}\,dx + \int_{-1}^{1} x\sin\frac{n\pi x}{2}\,dx + \int_{1}^{2} (2-x)\sin\frac{n\pi x}{2}\,dx\right]$

or $b_n = \left[\int_{0}^{1} x\sin\frac{n\pi x}{2}\,dx + \int_{1}^{2} (2-x)\sin\frac{n\pi x}{2}\,dx\right]$, (taking advantage of the fact that the product $f(x)\sin\frac{n\pi x}{2}$ is an even function).

12. $u(x,t) = 2e^{\frac{-5\pi^2 t}{16}}\sin\frac{\pi x}{4} - e^{-5\pi^2 t}\sin \pi x$