MATH 251
Final Exam
December 14, 2011

ANSWER KEY (Form A)

1. B
2. C
3. D
4. A
5. A
6. D
7. B
8. D
9. A
10. C
11. C
12. D
13. C
14. A
15. (a) T (b) F (c) F (d) F (e) T

16. (a) Eigenvalues are $\lambda = \frac{1}{4}, \quad \frac{9}{4}, \quad \frac{25}{4}, \ldots \quad , \quad \frac{(2n-1)^2}{4}; \qquad n = 1, 2, 3, \ldots$

Eigenfunctions are $X_n = \cos\frac{(2n-1)x}{2}; \qquad n = 1, 2, 3, \ldots$

(b) No, 0 is not an eigenvalue. $X = 0$, the trivial solution, is the only solution satisfies the boundary conditions. There is not a nonzero solution / eigenfunction.

17. (b) choice (*ii*)
(c) They both approach 1.
(e) $f(11)$ approaches 2. (With a period of 2, it reaches the same limit as $f(1)$.)

18. (a) $u(x,t) = 16 - 25e^{-12t}\cos(2x) - 36e^{-75t}\cos(5x)$
(b) $\lim_{t\to\infty} u(\pi,t) = 16$

(c) It will be higher. In the insulated-end case, the steady-state temperature is going to be equal to the constant term of the initial condition (as expanded into a Fourier cosine series). So the new limiting (steady-state) temperature will be 32 instead of 16.