

ANSWERS:

1. B
2. B or C
3. C
4. A
5. B
6. A
- 7.

(a) Critical points are $(1, 1)$ and $(1, -1)$
 (b) $(1, 1)$ is unstable (a saddle point); $(1, -1)$ is unstable (a spiral point/spiral source)

8.
$$y(t) = -\frac{7}{6}e^{-3t} + \frac{1}{2}e^t - \frac{1}{3}t + \frac{2}{3}$$

9. $m = 2, \gamma = 6, k = \frac{mg}{L} = 4$
 (a) $2u'' + 6u' + 4u = 0$ or $u'' + 3u' + 2u = 0, \quad u(0) = 0, u'(0) = 5$
 Solution: $u(t) = 5e^{-t} - 5e^{-2t}$

(b) (Trick question!) The system is overdamped. There is no oscillation, therefore, there is no quasi-period. Also acceptable: quasi-period = ∞ .

10.
 (a) $y = -3, 0, 3$
 (b) $y = -3$ is asymptotically stable; $y = 0$ is unstable; $y = 3$ is asymptotically stable.

11.

$$f(t) = 1 - u_{\frac{\pi}{2}}(t) + u_{\frac{\pi}{2}}(t) \sin 2t$$

$$F(s) = \mathcal{L}\{f(t)\} = \frac{1}{s} - \frac{e^{-\frac{\pi}{2}s}}{s} - \frac{2e^{-\frac{\pi}{2}s}}{s^2 + 4}$$

12.

$$a_0 = \frac{4\pi^2}{3}; \quad a_n = (-1)^{n+1} \frac{4}{n^2}, \quad n = 1, 2, 3, \dots$$

$$f(x) = \frac{2\pi^2}{3} + 4 \sum_1^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx$$

13. $\alpha^2 = 36, L = 4$

$$u(x, t) = e^{-36\pi^2 t} \sin(\pi x) - e^{-\frac{225}{4}\pi^2 t} \sin\left(\frac{5\pi}{4}x\right)$$

14.

$$X(t) = \begin{bmatrix} 3e^t + 2te^t \\ e^t + te^t \end{bmatrix}$$