# Answers to Fall 2000 Final exam.

**Q#:1**

$$y(t) = e^{2t}\left(\frac{t^3}{3} - 1\right)$$

**Q#:2**

**A** $y(t) = c_1 e^{-2t}\cos t + c_2 e^{-2t}\sin t$
**B** $y(t) = c_1 e^{3t} + c_2 t e^{3t}$
**C** $y(t) = c_1 e^{4t} + c_2 e^{-5t}$

**Q#:3**

$$y_p(t) = At\cos 3t + Bt\sin 3t + Cte^t + De^t + Et^2 + Ft + G$$

**Q#:4**

$$y(t) = \frac{3}{2}\sin 2t + \frac{1}{2}u_\pi(t)\sin(2t - 2\pi) = \frac{3}{2}\sin 2t + \frac{1}{2}u_\pi(t)\sin 2t$$

**Q#:5**

**A** $X(t) = c_1 \begin{bmatrix}-4\\1\end{bmatrix} e^{-4t} + c_2 \begin{bmatrix}1\\2\end{bmatrix} e^{5t}$
**B** Saddle point, which is unstable.
**C**
**D** $X(t) = \frac{1}{9}\begin{bmatrix}-4\\1\end{bmatrix} e^{-4t} - \frac{5}{9}\begin{bmatrix}1\\2\end{bmatrix} e^{5t}$

**Q#:6**

**A** The critical points are $P = (2,2)$ and $Q = (-2,2)$.
**B**

- Point P: $X' = \begin{bmatrix}0 & 1\\4 & -4\end{bmatrix}X$. $\lambda_1 = -2 + 2\sqrt{2}$ and $\lambda_2 = -2 - 2\sqrt{2}$.
- Point Q: $X' = \begin{bmatrix}0 & 1\\-4 & -4\end{bmatrix}X$. $\lambda = -2$.

**B**

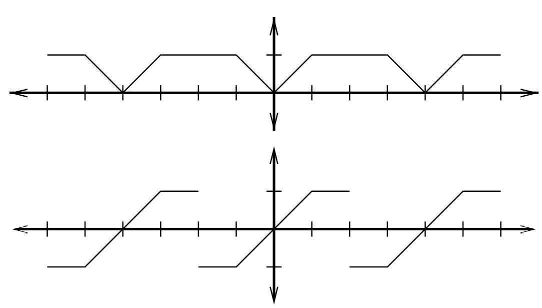
- Point P is a Saddle point, which is unstable.
- Point Q is an improper stable node, which is stable.

**Q#:7**

$$u(x,t) = \exp[\frac{-9\pi^2 t}{25}]\sin = frac2\pi x5 - 2\exp[-9\pi^2 t]\sin \pi x + 13\exp[\frac{-19(49)\pi^2 t}{25}]\sin\frac{7\pi x}{5}$$

**Q#:8**

A



**B** The even Fourier series is $f(x) = \frac{a_o}{2} + \sum_{i=1}^{\infty} a_n \cos\frac{n\pi x}{2}$ and the odd Fourier series is $f(x) = \sum_{i=1}^{\infty} b_n \sin\frac{n\pi x}{2}$.
**C** At $x = 2$ the even Fourier series converges to 1 and the odd Fourier serier converges to 0.

**Q#:9** $x^2 G''(x) + \lambda^2 G(x) = 0$ with boundary conditions $G(0) = G(1) = 0$.