MATH 251
Exam 2
July 27, 2010

ANSWER KEY

1.
$$\begin{cases} x_1' &= x_2 \\ x_2' &= x_3 \\ x_3' &= x_4 \\ x_4' &= -3tx_1 + x_2 - \frac{t^2}{2}x_4 + \frac{3}{2}\sin t \end{cases}$$

2. $g(3) = 10$

3. $F(s) = e^{-s}\dfrac{2s-6}{(s^2-6s+10)^2}$

4. $f(t) = u_3(t)(t - 4 + e^{-t+3})$

5. $f(t) = (1 - u_3(t))\,(2t^2 + t) + u_3(t)\,e^{4t} = 2t^2 + t + u_3(t)(\,e^{4t} - 2t^2 - t);$

$$F(s) = \frac{4}{s^3} + \frac{1}{s^2} + e^{-3s}\left(\frac{e^{12}}{s-4} - \frac{4}{s^3} - \frac{13}{s^2} - \frac{21}{s}\right)$$

6. (a) $\mathscr{L}\{y(t)\} = \dfrac{e^{-3s}}{s(s^2+4s+13)} + \dfrac{s+6-e^{-10s}}{s^2+4s+13}$

(b) $y(t) = 2e^{-2t}\cos 3t - 3e^{-2t}\sin 3t$
$$+u_{10}(t)\left(\frac{1}{13}e^{-2t+20}\cos(3t-30) + \frac{2}{39}e^{-2t+20}\sin(3t-30) - \frac{1}{13}\right)$$

7. (a) $x(t) = \begin{pmatrix} 2 \\ 4 \end{pmatrix} e^{2t} + \begin{pmatrix} -3 \\ 1 \end{pmatrix} e^{-5t}$

(b) $\alpha = 0$

(c) It is a *saddle point*, unstable.

8. (a) $x(t) = \begin{pmatrix} 4\sin 3t \\ 2\cos 3t + 2\sin 3t \end{pmatrix}$

(b) It is a *center*; (neutrally) stable.

9. (a) $x(t) = C_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^t + C_2 \left(\begin{bmatrix} 2 \\ 1 \end{bmatrix} t + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) e^t$

(b) It is an *improper node*, unstable

10. (a) The other critical points are $(0,0)$, and $(1/2, -1/2)$.

(b) $A = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$

(c) It is a *saddle point*, unstable.