MATH 251
Exam 2
July 28, 2008

ANSWER KEY

1. B

2. A

3. (a) $k = 8$; (b) $k = 4$; (c) none (since there is nonzero damping, resonance could never happen); (d) $T = \pi$.

4. (a) $f(t) = 2 + \cos 2t - \sin 2t$

(b) $f(t) = u_3(t)(4e^{3t-9}\cos(4t - 12) + 4.5e^{3t-9}\sin(4t - 12))$

5. $f(t) = (1 - u_5(t))(1 + 2t^2) + u_5(t)(e^{-4t} - t) = 1 + 2t^2 + u_5(t)(e^{-4t} - t - 1 - 2t^2)$;

$$F(s) = \frac{1}{s} + \frac{4}{s^3} + e^{-5s}\left(\frac{e^{-20}}{s+4} - \frac{4}{s^3} - \frac{21}{s^2} - \frac{56}{s}\right)$$

6. $y(t) = u_5(t)\left(\frac{-2}{9} + \frac{2}{9}e^{-3t+15} + \frac{2}{3}(t-5)e^{-3t+15}\right)$

7. (a) $x(t) = \begin{pmatrix} 9 \\ 3 \end{pmatrix} e^{4t} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t}$

(b) It is a *saddle point*, unstable.

8. (a) $x(t) = C_1\begin{pmatrix} \cos 4t + 3\sin 4t \\ 2\cos 4t \end{pmatrix} + C_2\begin{pmatrix} -3\cos 4t + \sin 4t \\ 2\sin 4t \end{pmatrix}$

(b) It is a *center*; (neutrally) stable.

(c) $x(t) = C_1\begin{pmatrix} 4 \\ 0 \end{pmatrix} e^{-7t} + C_2\begin{pmatrix} -1 \\ 3 \end{pmatrix} e^{-7t}$

(d) It is a *proper node* (star point); asymptotically stable.

9. (a) The critical points are $(-1, 1)$, $(-1, 0)$, and $(-2, 2)$.

(b) It is a *node*, unstable.