MATH 251
Exam 2
July 30, 2007

ANSWER KEY

**1.** (a) $k < 12$; (b) critically damped; (c) $k = 15$; (d) $T = 2\pi$; (e) true.

**2,** (a) $k = 50$

(b) $2u'' + 50u = 0,\quad u(0) = 1,\quad u'(0) = 4$

**3.** D

**4.** (a) $f(t) = -1 + 2e^{-t} - 2te^{-t}$

(b) $f(t) = u_4(t)(2e^{-t+4}\cos(5t - 20) - 2e^{-t+4}\sin(5t - 20))$

**5.** $f(t) = (1 - u_4(t))\, e^{2t} + u_4(t)(8t - t^2) = e^{2t} + u_4(t)(8t - t^2 - e^{2t})$;

$$F(s) = \frac{1}{(s-2)} + e^{-4s}\left(\frac{-2}{s^3} + \frac{16}{s} - \frac{e^8}{s-2}\right)$$

**6.** $$y(t) = \frac{2}{7}e^{t} - \frac{2}{7}e^{-6t} + u_3(t)\left(\frac{1}{7}e^{t-3} - \frac{1}{7}e^{-6t+18}\right)$$

**7.** $$Y(s) = \frac{1}{(s+2)(s^3 - 2s^2 + s + 4)} + \frac{e^{-\pi s}}{s^3 - 2s^2 + s + 4} + \frac{s-2}{s^3 - 2s^2 + s + 4}$$

**8.** (a) $x(t) = C_1\begin{pmatrix} -6 \\ 4 \end{pmatrix}e^{-2t} + C_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{8t}$

(b) $\beta = -4$

**9.** (a) There are two linearly independent eigenvectors for the eigenvalue 2. Therefore, it is a *proper node* (star point), and it is unstable.

(b) $x(t) = C_1 e^{-4t}\begin{pmatrix} \cos 5t - \sin 5t \\ 2\sin 5t \end{pmatrix} + C_2 e^{-4t}\begin{pmatrix} \cos 5t + \sin 5t \\ -2\cos 5t \end{pmatrix}$

(c) It is a *spiral point*; asymptotically stable.

(d) It is a *saddle point*; unstable.