

MATH 251
Summer 2006
Exam II
July 20, 2006

ANSWERS:

1. (a) $T = \pi$ (seconds); (b) $\gamma = 8$; (c) Underdamped; (d) True.

2. D

3. C

4. B

5. D

6. (a) $Y = (A_0t^3 + A_1t^2 + A_2t)e^t + A_4e^{3t} \cos 2t + A_5e^{3t} \sin 2t + (A_6t + A_7) \cos 3t + (A_8t + A_9) \sin 3t$;

(b) $y(t) = C_1e^t + C_2e^{3t} + t^2 + \frac{8}{3}t + \frac{23}{9}$.

7. (a) The required initial value problem is: $2u'' + 4u' + 10u = 0$, $u(0) = 4$, $u'(0) = 0$;
its solution is $u(t) = 4e^{-t} \cos 2t + 2e^{-t} \sin 2t$.

(b) 2 (rad/sec)

(c) $\lim_{t \rightarrow \infty} u(t) = 0$

8. $f(t) = (u_2(t) - u_4(t))(3t - 6) + u_4(t)(4e^{-3t}) = u_2(t)(3t - 6) + u_4(t)(4e^{-3t} - 3t + 6)$; its Laplace transform is $F(s) = e^{-2s} \frac{3}{s^2} + e^{-4s} \left(\frac{4e^{-12}}{s+3} - \frac{3}{s^2} - \frac{6}{s} \right)$.

9. $y(t) = \frac{-1}{2}e^{-2t} \sin 2t + \frac{1}{2}u_\pi(t)e^{-2(t-\pi)} \sin(2t - 2\pi)$.

10. (a) $\mathbf{x}(t) = \begin{pmatrix} 3 \\ 3 \end{pmatrix} e^{-t} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-4t}$.

(b) $\lim_{t \rightarrow \infty} |\mathbf{x}(t)| = 0$.