MATH 251
Fall 2002
Exam 2
November 12, 2002

ANSWERS:

1. For $y_1 = y$ and $y_2 = y'$ we have:

$$\begin{cases} x^2 y_2' + x y_2 + (x^2 - \lambda^2) y_1 = 0 \\ y_1' = y_2 \end{cases} \qquad y_1(0) = y_0, \qquad y_2(0) = y_0'$$

Simplifying:

$$\begin{cases} y_1' = y_2 \\ y_2' = \frac{-1}{x^2}(x^2 - \lambda^2) y_1 - \frac{1}{x} y_2 \end{cases} \qquad y_1(0) = y_0, \qquad y_2(0) = y_0'$$

2. $Y = Ae^{2t} + t(B\sin(2t) + C\cos(2t)) + D\sin t + E\cos t + Ft^2 + Gt + H$

3.(b) $f(t) = t^2 + (6 - t - t^2)u_2(t) + (t-6)u_6(t)$

4. It is critically damped, so $\gamma^2 - 4km = 0$. Therefore, $\gamma = \sqrt{4km} = 12$.

5. $\mathcal{L}\{y(t)\} = s^3 \dfrac{3}{(s-\pi)^2 + 9} - 3s - 6\pi$

6. $y(t) = \dfrac{1}{2}e^{2t} + \dfrac{1}{2}e^{-4t} + (\dfrac{1}{7}e^{3(t-3)} - \dfrac{1}{6}e^{2(t-3)} + \dfrac{1}{42}e^{-4(t-3)})e^9 u_3(t)$

7. $y(t) = e^{-4t} + u_3(t)(\dfrac{2}{3}e^{-(t-3)} - \dfrac{2}{3}e^{-4(t-3)})$;

8. $X(t) = \begin{pmatrix} 3 \\ -3 \end{pmatrix} e^{-3t} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} e^{4t}$

9.(a) $X(t) = C_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t \cos(3t) + C_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^t \sin(3t)$

(b) $X(t) = C_1 e^t \begin{pmatrix} 1 \\ -1 \end{pmatrix} + C_2 e^t \begin{pmatrix} t + \frac{1}{2} \\ -t \end{pmatrix}$