1. $y_2(t) = t^{-2}$ (or any of its nonzero constant multiples).

2. a) The homogeneous solution is $y_c(t) = C_1e^{3t} + C_2te^{3t}$. The right-hand side of the equation is $3e^{2t}$. The form of the particular solution is, therefore, $Y(t) = Ae^{2t}$. Substitute $Y(t)$ into the equation and solve: $Y'' - 6Y' + 9Y = 3e^{2t}$. The result is $A = 3$, so $Y(t) = 3e^{2t}$. Finally, the solution is $y(t) = y_c(t) + Y(t) = C_1e^{3t} + C_2te^{3t} + 3e^{2t}$.

b) $Y(t) = (At + B)\cos 2t + (Ct + D)\sin 2t + Et^3 + Ft^2 + Gt + H + It^2e^{3t}$.

3. a) Since the Wronskian is $W(y_1(1), y_2(1)) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} t & te^t \\ 1 & te^t + e^t \end{vmatrix} = t^2e^t \neq 0$ when $t > 0$, the functions $y_1(t) = t$ and $y_2(t) = te^t$ do form a set of fundamental solutions.

b) Since y_1 and y_2 form a set of fundamental solutions, the general solution is $y(t) = C_1y_1 + C_2y_2 = C_1t + C_2te^t$.

4. Constants: $m = 2$, $g = 10$, $L = 1$; $mg = 20 = kL$ so $k = 1$; the resistive force is $F_r = \gamma|V| = 8$ when $|V| = 2$, so the damping constant is $\gamma = 4$. The initial displacement is 1, and the initial velocity is $3\sqrt{3} - 1$.

a) The required initial value problem for $u(t)$ is

$$2u'' + 4u' + 20u = 0, \qquad u(0) = 1, \quad u'(0) = 3\sqrt{3} - 1.$$

Its solution is $u(t) = e^{-t}\cos 3t + \sqrt{3}e^{-t}\sin 3t$.

b) $\lim_{t\to\infty} u(t) = 0$

c) The system is underdamped, therefore, the mass will oscillate (i.e., pass through the equilibrium position) infinitely many times.

5. a) $y(t) = e^{2t}\cos 3t + 2e^{2t}\sin 3t$.

b) $f(t) = (1 - u_{2\pi}(t))(3t^2) + u_{2\pi}(t)(3t^2 + \sin t) = 3t^2 + u_{2\pi}(t)\sin t$.

$F(s) = \mathcal{L}\{3t^2\} + e^{-2\pi s}\mathcal{L}\{\sin(t + 2\pi)\} = \dfrac{6}{s^3} + e^{-2\pi s}\mathcal{L}\{\sin t\} = \dfrac{6}{s^3} + e^{-2\pi s}\dfrac{1}{s^2 + 1}$.

6. $y(t) = \dfrac{1}{4}e^{2t} - \dfrac{1}{4}e^{-2t} + u_{2\pi}(t)[\dfrac{1}{4}e^{2(t-2\pi)} - \dfrac{1}{4}e^{-2(t-2\pi)}]$