Math 251 Spring 2002 Midterm Exam 1

1. (b)

2. (d) because between equilibrium solutions $y = 0$ and $y = 1$ the right-hand side of the equation is negative, therefore the solution starting at $\frac{9}{10}$ must decrease to a nearest equilibrium solution, which is 0.

3. (d) since we need to check when

$$\frac{\partial}{\partial y}(e^x \sin y + bx^2y^2) = \frac{\partial}{\partial x}(e^x \cos y + x^3 y)$$

4. (c)

5. (d) because after dividing the whole equation by $t^2 - 1$ we will get discontinuities of the coefficients at -1 and 1, and the initial point 5 lies to the right of the last point of discontinuity.

6.

$$\frac{dy}{dx} = -\frac{x}{y}, \qquad y(0) = -1$$

$$ydy = -xdx$$

$$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + const$$

$$y^2 = -x^2 + const$$

$$(-1)^2 = -0^2 + const$$

$$const = 1$$

$$\boxed{y(x) = -\sqrt{-x^2 + 1}}$$

7.

$$t^2\frac{dy}{dt} + 3ty = e^t, \qquad t > 0$$

$$\frac{dy}{dt} + \frac{3}{t}y = \frac{1}{t^2}e^t$$

$$\mu(t) = e^{\int \frac{3}{t}dt} = e^{3\ln t} = t^3$$

$$\boxed{y(t) = \frac{\int t^3 \frac{1}{t^2} e^t dt}{t^3} = \frac{te^t - e^t + C}{t^3} = \frac{1}{t^2}e^t - \frac{1}{t^3}e^t + \frac{C}{t^3}}$$

8. First not that the equation is exact. Now

$$\psi = \int (2xy - 3x^2)dx = x^2y - x^3 + C(y)$$

$$\frac{\partial}{\partial y}(x^2y - x^3 + C(y)) = x^2 + C'(y) = x^2 + 2y$$

$$C'(y) = 2y$$

$$C(y) = y^2 + const$$

$$\boxed{\psi = x^2y - x^3 + y^2 = const}$$

9. (a)

$$y'' - 4y' + 4y = 0$$
$$\lambda^2 - 4\lambda + 4 = 0$$
$$\lambda_{1,2} = 2$$
$$\boxed{y(t) = C_1 e^{2t} + C_2 t e^{2t}}$$

(b)

$$y'' - 4y' + 5y = 0$$
$$\lambda^2 - 4\lambda + 5 = 0$$
$$\lambda_{1,2} = 2 \pm i$$
$$\boxed{y(t) = C_1 e^{2t} \cos t + C_2 e^{2t} \sin t}$$

10.

$$y'' - 4y = 0, \qquad y(0) = 4, \qquad y'(0) = 4$$
$$\lambda^2 - 4 = 0$$
$$\lambda_1 = 2 \qquad \lambda_2 = -2$$
$$y(t) = C_1 e^{2t} + C_2 e^{-2t}$$
$$y(0) = C_1 + C_2 = 4$$
$$y'(0) = 2C_1 - 2C_2 = 4$$
$$C_1 = 3 \qquad C_2 = 1$$
$$\boxed{y(t) = 3e^{2t} + e^{-2t}}$$

11.

$$\frac{dQ}{dt} = 2 \cdot 5 - 2 \cdot \frac{Q(t)}{2}, \qquad Q(0) = 15$$
$$\frac{dQ}{dt} + Q(t) = 10$$
$$Q(t) = Ce^{-t} + 10$$
$$Q(0) = C + 10 = 15$$
$$C = 5$$
$$Q(t) = 5e^{-t} + 10$$
$$\boxed{Q(\ln 5) = 5e^{-\ln 5} + 10 = 11}$$

12.

$$m\frac{dv}{dt} = mg - 2v \qquad\qquad v(0) = 0$$

$$\frac{dv}{dt} + v = 10$$

$$v(t) = Ce^{-t} + 10$$

$$v(0) = C + 10 = 0$$

$$C = -10$$

$$v(t) = -10e^{-t} + 10$$

$$\boxed{\text{distance} = \int_0^2 v(t)dt = \int_0^2 (-10e^{-t} + 10)dt = \left.(10e^{-t} + 10t)\right|_0^2 = 10 + 10e^{-2}}$$