MATH 251
Exam 1
Feb 21, 2000

ANSWER KEY

1. (a) first order, non-linear, homogenerous, ordinary
(b) second order, linear, non-homogeneous, ordinary
(c) second order, non-linear, non-homogeneous, ordinary
(d) second order, linear, homogeneous, ordinary

2. (a) $\frac{dQ}{dt} = 6 - \frac{3Q}{100}$, or $\frac{dQ}{dt} + \frac{3Q}{100} = 6$; $Q(0) = 0$,

where $Q(t)$ = the amount of dissolved salt in the tank at time t.

(b) $Q(t) = 200 - 200e^{-3t/100}$

(c) The limiting concentration is $\lim_{t\to\infty} \frac{Q(t)}{100} = \frac{200}{100} = 2$.

3. $y(t) = t^{-2}(-t\cos t + \sin t + \frac{\pi^2 - 4}{4})$; the largest interval is $(0, \infty)$.

4. (a) $y(t) = \frac{-2}{3}e^{-t} + (y_0 + \frac{2}{3})e^{t/2}$

(b) $y_0 = -2/3$; at this value y would approach 0 as t approaches ∞.

5. (a) $y(t) = C_1 e^{2t} + C_2 e^{-t}$

(b) $y(t) = C_1 e^{-2t} + C_2 te^{-2t}$

(c) $y(t) = C_1 e^{-2t}\cos t + C_2 e^{-2t}\sin t$

6. This (undamped) system is described by the equation $2u'' + 8u = 0$. The system, therefore, has a natural frequency of $\omega_0 = \sqrt{k/m} = 2$ (radians/sec). Hence, resonance would occur if $\omega = \omega_0 = 2$.

7. (a) The 3 equilibrium solutions are: $y = 0$ (stable), $y = T$ (unstable), and $y = K$ (stable).

(b) The population would die out if $0 < y_0 < T$, its size would become stable if $y_0 \geq T$. The population would never increase without bound.

8. The complementary solution is $y_c(t) = C_1 e^{3t} + C_2 te^{3t}$. Therefore, the particular solution should have the form:

$Y(t) = A_1 t^2 + A_2 t + A_3 + A_4 t^2 e^{3t} + A_5 \cos t + A_6 \sin t$.