MATH 251
Fall 2001
Exam I
September 25, 2001

ANSWERS:

1. There are infinitely many correct answers for each part. A few examples are given.
(a) e.g., $y' = y^2$ or $y' = e^y$
(b) e.g., $y'' = 0$ or $y'' + y' + y = 0$

2. $y(t) = \sqrt{2e^t - 1}$

3. (First move everything to the left-hand side of the equation and check to see that this is an exact equation.)
Solution: $-2x - e^{xy} + y^2 = C$

4.
The initial value problem is $Q' = 8 - \frac{1}{50}Q, \qquad Q(0) = 100.$
The solution is $Q(t) = 400 - 300e^{\frac{-t}{50}}$.
Finally, $\lim_{t\to\infty} Q(t) = 400$

5. First rewrite the equation as $y' - \dfrac{3}{t}y = 1$, $y(4) = -1$.
(a) The guaranteed solution interval is $(0, \infty)$.
(b) $y(t) = \dfrac{-t}{3} + \dfrac{t^3}{64}$

6. (a) The equilibrium solutions are: $y = -1$ (stable), $y = 1$ (unstable), and $y = 2$ (stable).

7. (a) $y(t) = C_1e^{2t}\cos t + C_2e^{2t}\sin t$
(b) $y(t) = C_1e^{-3t} + C_2te^{-3t}$

8. (a) $y(t) = 3e^t - e^{4t}$
(b) $\lim_{t\to\infty} y(t) = \lim_{t\to\infty} e^t(3 - e^{3t}) = -\infty$

9. First substitute $y_1(t)$ and $y_2(t)$ into the equation to verify that they both satisfy it. This shows that both functions are indeed solutions of the given equation. Then calculate their Wronskian, $W(y_1, y_2) = t^4 \neq 0$ when $t > 0$. This shows that they are linearly independent. Therefore, the two functions do, in fact, form a fundamental set of solutions for the given equation.