

**ANSWERS:**

1. (a) \( F = a_0(t)y + a_1(t)y' + a_2(t)y'' + \ldots + a_n(t)y^{(n)} + g(t), \quad a_n(t) \neq 0. \)
   (b) (i) second order and linear; (ii) third order and non-linear; (iii) first order and linear.

2. (a) \( y(t) = \alpha e^{-t} - \alpha e^{-2t}. \)
   (b) All real numbers; all values of \( \alpha \) would make the limit 0.
   (c) \( \alpha = 0 \) only.

3. (a) \( y = 0 \) and \( y = 3. \)
   (b) \( y = 0 \) is a stable equilibrium solution, \( y = 3 \) is an unstable equilibrium solution.
   (c) Since \( y = 0 \) is an equilibrium solution, it’s a constant solution. Therefore, if \( y(0) = 0 \) then \( y(1) = 0. \)

4. (a) \( y(t) = \frac{1}{\sin t}[-3t \cos t + 3 \sin t - 3] = -3t \cot t + 3 - 3 \csc t. \)
   (b) \((0, \pi)\) is the largest interval.

5. (a) \( y^2 = \frac{x^3}{3} + C. \)
   (b) \( \frac{x^2}{2} - xy + \frac{y^2}{2} = C, \quad \text{or (after first dividing both sides by } x - y), \quad y = x + C. \)

6. (a) \( Q' = -\frac{20}{1100}Q, \quad \text{or} \quad Q' + \frac{1}{55}Q = 0; \quad Q(0) = 100 \)
   where \( Q(t) \) is the amount of blue M&Ms in the vat at time \( t. \)
   (b) \( Q(t) = 100e^{-\frac{10}{11}t}. \)
   (c) \( t = 55 \ln\left(\frac{100}{11}\right). \)