MATH 251
Fall 2000
Exam I
September 26, 2000

ANSWERS:

1.
(a) $F = a_0(t)y + a_1(t)y' + a_2(t)y'' + ... + a_n(t)y^{(n)} + g(t), \qquad a_n(t) \neq 0.$
(b) (i) second order and linear; (ii) third order and non-linear; (iii) first order and linear.

2.
(a) $y(t) = \alpha e^{-t} - \alpha e^{-2t}$.
(b) All real numbers; all values of α would make the limit 0.
(c) $\alpha = 0$ only.

3.
(a) $y = 0$ and $y = 3$.
(b) $y = 0$ is a stable equilibrium solution, $y = 3$ is an unstable equilibrium solution.
(c) Since $y = 0$ is an equilibrium solution, it's a constant solution. Therefore, if $y(0) = 0$ then $y(1) = 0$.

4.
(a) $y(t) = \dfrac{1}{\sin t}[-3t\cos t + 3\sin t - 3] = -3t\cot t + 3 - 3\csc t.$
(b) $(0, \pi)$ is the largest interval.

5.
(a) $y^2 = \dfrac{x^3}{3} + C.$
(b) $\dfrac{x^2}{2} - xy + \dfrac{y^2}{2} = C, \qquad$ or (after first dividing both sides by $x - y$), $\qquad y = x + C.$

6. (a) $Q' = -\dfrac{20}{1100}Q, \qquad$ or $\qquad Q' + \dfrac{1}{55}Q = 0; \qquad Q(0) = 100$
where $Q(t)$ = the amount of blue M&Ms in the vat at time t.
(b) $Q(t) = 100e^{-\frac{t}{55}}$.
(c) $t = 55\ln(\dfrac{100}{11})$.