MATH 140A
Spring 2004
Final Examination
May 3, 2004

ANSWERS:

1. E
2. A
3. A
4. D
5. E
6. B
7. B
8. B
9. D
10. A
11. D
12. C
13. E
14. D
15. B
16. B
17.
a) T b) T c) F d) T e) F f) F g) T h) F

18.
Let $u = 2 + \sqrt{x}$ then $du = \dfrac{1}{2\sqrt{x}}dx$, and $\dfrac{1}{4\sqrt{x}}dx = \dfrac{1}{2}du$
Limits of integration: when $x = 0$, $u = 2$; and when $x = 4$, $u = 4$.
After substitution, the integral becomes:
$$\int_2^4 \frac{1}{2}u^3\,du = \frac{1}{8}u^4\bigg|_2^4 = \frac{1}{8}(256-16) = \frac{1}{8}(240) = 30$$

19.
a) The intersections are $(0,0)$, $(\sqrt{3}, 3\sqrt{3})$, and $(-\sqrt{3}, -3\sqrt{3})$.
c) $Area = \displaystyle\int_{-\sqrt{3}}^{0}(x^3-3x)\,dx + \int_0^{\sqrt{3}}(3x-x^3)\,dx \qquad OR \qquad = 2\int_0^{\sqrt{3}}(3x-x^3)\,dx$ (due to symmetry)
$$= \left(\frac{x^4}{4} - \frac{3x^2}{2}\right)\bigg|_{-\sqrt{3}}^{0} + \left(\frac{3x^2}{2} - \frac{x^4}{4}\right)\bigg|_0^{\sqrt{3}} = \left[0 - \left(\frac{9}{4} - \frac{9}{2}\right)\right] + \left[\left(\frac{9}{2} - \frac{9}{4}\right) - 0\right] = \frac{9}{4} + \frac{9}{4} = \frac{9}{2}$$

20.
a) $V = \displaystyle\int_0^{\sqrt{3}} \pi[(3x)^2 - (x^3)^2]\,dx = \int_0^{\sqrt{3}} \pi(9x^2 - x^6)\,dx$ (Washer method)

OR $V = \displaystyle\int_0^{3\sqrt{3}} 2\pi y(y^{1/3} - \frac{1}{3}y)\,dy = 2\pi\int_0^{3\sqrt{3}}(y^{4/3} - \frac{1}{3}y^2)\,dy$ (Shell method)

b) $V = \displaystyle\int_0^{\sqrt{3}} 2\pi x(3x - x^3)\,dx = 2\pi\int_0^{\sqrt{3}}(3x^2 - x^4)\,dx$ (Shell method)

OR $V = \displaystyle\int_0^{3\sqrt{3}} \pi((y^{1/3})^2 - (\frac{1}{3}y)^2)\,dy = \int_0^{3\sqrt{3}} \pi(y^{2/3} - \frac{1}{9}y^2)\,dy$ (Washer method)

(You only need to set up the integrals. Simplifying the expressions is optional.)