

MATH 140A  
Spring Semester 2002  
Exam III  
April 1, 2002

**ANSWERS:**

1. B; 2. A; 3. D; 4. E; 5. A; 6. D; 7. E; 8. C.

9. A critical number is (the  $x$ -coordinate of) an interior point in the domain of a function where the derivative is either zero or is undefined (i.e. where  $f'(x) = 0$  or where  $f'(x)$  does not exist).

10.

a.  $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

b.  $x = 0$  ( $x = 3$  and  $x = -3$  are NOT critical numbers since they are not in the domain; in fact, they are the vertical asymptotes.)

c.  $y = 0$  (i.e., the  $x$ -axis)

d.  $x = 3$  and  $x = -3$

e.  $(-\infty, -3) \cup (-3, 0)$

f.  $(0, 3) \cup (3, \infty)$

g. 0

h. There is none.

i.  $(-\infty, -3) \cup (3, \infty)$

j.  $(-3, 3)$

k. There is none.

11.  $h$  is always negative, always increasing, and always concave down.

12.

$$f = ax^3 + bx^2 + cx + d$$

$$f' = 3ax^2 + 2bx + c$$

$$f'' = 6ax + 2b$$

Where  $a$  cannot be zero, else this is not a third degree polynomial.

a. No critical numbers means that  $f'(x)$  must have two complex roots, so its discriminant  $(2b)^2 - 4(3a)(c)$  must be negative. Take, for example,  $a = c = 1$  and  $b = 0$  ( $d$  can be anything, so set it to 0):

$$f(x) = x^3 + x$$

b. Two critical numbers means that  $f'(x)$  must have two distinct real roots, so its discriminant  $(2b)^2 - 4(3a)(c)$  must be positive. Take, for example,  $a = 1$ ,  $b = 1$  and  $c = 0$  ( $d$  can be anything, so set it to 0):

$$f(x) = x^3 + x^2$$

c. The second derivative  $f''$  must be zero at  $x = 3$ , so  $(x - 3)$  must be a factor of  $f'' = 6ax + 2b$ . That's  $2b$  has to be  $-3$  times  $6a$ , or  $b = -9a$ .  $f''$  also must be negative when  $x < 3$  and positive when  $x > 3$ . With that in mind, set  $a = -1$  and  $b = 9$  in  $f$  ( $c$  and  $d$  can be anything, so set them to 0):

$$f(x) = -x^3 + 9x^2$$

( $f(x) = x^3 - 9x^2$  would have the wrong concavity.)

d. The first derivatives must be zero at  $x = 0$  and  $x = 2$ , so  $x$  and  $(x - 2)$  must both be factors of  $f'$ . Since  $f'$  is already a quadratic polynomial, it must be a constant multiple of  $x(x - 2)$ . That is,  $f'(x) = k(x^2 - 2x) =$

$kx^2 - 2kx$ , for some nonzero number  $k$  (can let it to be 1, for now). Equate this with  $f' = 3ax^2 + 2bx + c$  we see that  $c$  must be zero and  $2b$  is  $-2$  times  $3a$ , or  $b$  is  $-3$  times  $a$ . Setting  $a = 1$ ,  $b = -3$  and  $c = 0$ , we get  $f(x) = kx^3 - 3kx^2$ . But, checking the signs and we see that  $k$  must be a negative number, so set  $k = -1$  (again,  $d$  can be anything, so set it to 0):

$$f(x) = -x^3 + 3x^2$$

( $f(x) = x^3 - 3x^2$  would have a maximum at  $x = 0$  and a minimum at  $x = 2$ .)

13. The desired radius is  $r = \frac{1}{2}V^{\frac{1}{3}}$

14. The largest possible area is 4.