Name:______________________

MATH 251
Final exam
May 4, 2000

Section:______________________

There are **10** partial credit questions, each worth 15 points. **In order to obtain full credit for these problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is in parentheses to the right of the question number.

YOU MAY NOT USE A CALCULATOR ON THIS EXAM.

1:________
2:________
3:________
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7:________
8:________
9:________
10:________
Total:________

Do not write in the box to the left

1. Solve the initial value problem

$$y' = 2t + 2ty, \quad y(0) = 3.$$

2. Find the general solution to each of the following:

   (a) $4y'' - 4y' + y = 0$

   (b) $y'' - 2y' + 2y = 0$

   (c) $y'' + 5y' + 6y = 0$

3. Find the *form* of a particular solution of the equation:

$$y'' + 4y = t^2 e^t - 2\cos 2t + (t+1)\sin t.$$

DO NOT solve for the constants.

4. (a) Find the Laplace transform of $f(t) = \begin{cases} 0 & : 0 \leq t < 3 \\ t^2 & : t \geq 3 \end{cases}$.

(b) Find the inverse Laplace transform of $\dfrac{s-2}{s^2+2s+10}$.

5. Solve the following initial value problem.

$$y'' - 2y' = 4\delta(t-1), \quad y(0) = 0, \quad y'(0) = 0$$

6. Let $\mathbf{A} = \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}$.

   (a) Find the general solution of $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

   (b) Classify the type and stability of the critical point at $\binom{0}{0}$

   (c) If $\mathbf{x}(0) = \begin{pmatrix} 2 \\ \alpha \end{pmatrix}$ and $\lim_{t \to \infty} \mathbf{x}(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, then what is the value of α?

7. Consider the following nonlinear system:

$$x' = x^2 + y^2 - 10$$
$$y' = 2x - 6y$$

   (a) Find all the critical (fixed) points.

   (b) For each critical point, find the eigenvalues of the linearized system near that point.

   (c) What conclusions can you draw about the type and stability of the critical points of the nonlinear system?

8. Separate the following partial differential equation into two ordinary differential equations.

$$2u_{xx} + u_{xt} = 0$$

9. Solve the heat conduction equation,

$$4u_{xx} = u_t$$
$$u(0,t) = 0 = u(3,t) \quad \text{for } t > 0$$
$$u(x,0) = \sin(\frac{2\pi x}{3}) - 2\sin(\pi x) + 7\sin(\frac{5\pi x}{3})$$

10. Let

$$f(x) = \begin{cases} 0 & : 0 \leq x < 1 \\ 1 & : 1 \leq x < 2. \end{cases}$$

(a) Sketch the even, period 4 extension of $f(x)$ on the interval $-6 \leq x \leq 6$.

(b) Find the Fourier series for the above extension.

(c) What does the above series converge to when $x = 0$, $x = -3/2$, $x = 1$, and $x = 2$?

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	
1	$\dfrac{1}{s},$	$s > 0$
e^{at}	$\dfrac{1}{s-a},$	$s > a$
t^n, n a positive integer	$\dfrac{n!}{s^{n+1}},$	$s > 0$
t^p, $p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}},$	$s > 0$
$\sin at$	$\dfrac{a}{s^2+a^2},$	$s > 0$
$\cos at$	$\dfrac{s}{s^2+a^2},$	$s > 0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2},$	$s > a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2},$	$s > a$
$t^n e^{at}$, n a positive integer	$\dfrac{n!}{(s-a)^{n+1}},$	$s > a$
$u_c(t)$	$\dfrac{e^{-cs}}{s},$	$s > 0$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$	
$e^{ct}f(t)$	$F(s-c)$	
$f(ct)$, $c > 0$	$\frac{1}{c}F(\frac{s}{c})$	
$\delta(t-c)$	e^{-cs}	
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	