Name:________________

MATH 251 — Student Number:________________

Final exam — Instructor:________________

Dec. 15, 2000 — Section:________________

There are **9** partial credit questions. **In order to obtain full credit for these problems, all work must be shown. Credit will not be given for an answer not supported by work.**

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION. At the end of the examination, the booklet will be collected.

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Do not write in the box to the left

1. (16 pts) Solve the initial value problem

$$y' - 2y = t^2 e^{2t}, \qquad y(0) = -1.$$

2. (16 pts) Find the general solution to each of the following:
   (a) $y'' + 4y' + 5y = 0$
   (b) $y'' - 6y' + 9y = 0$
   (c) $y'' + y' - 20y = 0$

3. (16 pts) Find the *form* of a particular solution of the equation:

$$y'' + 9y = 7\sin 3t + 2te^t - t^2.$$

DO NOT solve for the constants!

4. (16 pts) Solve the following initial value problem:

$$y'' + 4y = \delta(t - \pi), \qquad y(0) = 0, \qquad y'(0) = 3.$$

5. (18 pts) Let $A = \begin{pmatrix} -3 & 4 \\ 2 & 4 \end{pmatrix}$.

   (a) Find the general solution of $\mathbf{x}' = A\mathbf{x}$.

   (b) Classify the type and stability of the critical point at the origin.

   (c) Sketch the phase portrait.

   (d) Find the particular solution satisfying $\mathbf{x}(0) = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$.

6. (20 pts) Consider the following nonlinear system:

$$\begin{aligned} x' &= y - 2 \\ y' &= x^2 - y^2 \end{aligned}$$

(a) Find all the critical points.

(b) Linearize the system near each critical point and find the eigenvalues of the resulting linear system.

(c) What can you conclude about the type and stability of the critical points of the nonlinear system?

7. (16 pts) Solve the heat conduction problem described by:

$$\begin{aligned} u_t &= 9u_{xx}, \qquad 0 < x < 5, \quad t > 0 \\ u(0,t) &= u(5,t) = 0 \quad \text{for} \quad t > 0 \\ u(x,0) &= \sin\tfrac{2\pi x}{5} - 3\sin \pi x + 13\sin\tfrac{7\pi x}{5} \end{aligned}$$

8. (16 pts) Let

$$f(x) = \begin{cases} x & 0 \le x < 1 \\ 1 & 1 \le x < 2 \end{cases}$$

(a) Sketch the graphs of the even and odd extensions of f of period 4, for three periods; state which graph corresponds to which extension.

(b) Which extension has a Fourier sine series, and which has a cosine series? Write down the *form* of each series, but **DO** NOT compute the coefficients.

(c) To what value does each series converge at $x = 2$?

9. (16 pts) Consider the following initial/boundary value problem:

$$\begin{aligned} tu_t &= x^2 u_{xx} \\ u(0,t) &= u(1,t) = 0 \quad \text{for} \quad t > 0 \\ u(x,0) &= f(x) \end{aligned}$$

Separate the variables by setting $u(x,t) = X(x)T(t)$ and state (but **D**O NOT SOLVE!) the resulting eigenvalue problem for X.