

MATH 251
Summer 2003
Exam 2
July 24, 2003

NAME : _____

ID : _____

INSTRUCTOR : _____

There are 9 questions on 8 pages.

Please read each problem carefully before starting to solve it. For each multiple choice problem 4 answers are given, only one of which is correct. Mark only one choice.

For partial credit questions, all work must be shown - **credit will not be given for an answer unsupported by work.**

NO CALCULATORS ARE ALLOWED.

PLEASE DO NOT WRITE IN THE BOX BELOW.

A TABLE OF LAPLACE TRANSFORMS IS ATTACHED.

1: _____
2: _____
3: _____
4: _____
5: _____
6: _____
7: _____
8: _____
9: _____
Total: _____

1. (5 points) If a mass-spring system described by the equation $2u'' + ku = 10 \cos 4t$ exhibits resonance, then what is the value of k ?

- (a) 4
- (b) 8
- (c) 16
- (d) 32

2. (5 points) The inverse Laplace transform of $\frac{8}{s^3 - 4s}$ is

- (a) $-2 + e^{2t} + e^{-2t}$
- (b) $-2t + e^{2t} + e^{-2t}$
- (c) $2t + \cos 2t$
- (d) $-2 + \frac{1}{2} \sin 2t$

3. (6 points) True or false:

- i. $\mathcal{L}\{f(t)g(t)\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$, T F
- ii. $\mathcal{L}\{u_\pi(t) \sin t\} = e^{-\pi s} \frac{1}{s^2 + 1}$, T F
- iii. If $f(t) = 2 - u_2(t) + 3u_4(t)$, then $f(3) = 1$, T F

4. (16 points)

- (a) When using the method of undetermined coefficients to solve the following non-homogeneous equation, what is the form of the particular solution? (Do not solve for the coefficients!)

$$y'' + 4y' + 4y = 3t^2 - te^{-2t} + 2t \cos 3t.$$

- (b) Find the general solution of

$$y'' - y = 3e^{-t}.$$

5. (14 points) A mass of 1 kg stretches a spring 50 cm . The mass-spring system has damping of $\gamma = 4 \text{ N} \cdot \text{sec}/\text{m}$. At $t = 0$ the mass is set in motion from its equilibrium position with downward velocity of $2 \text{ m}/\text{sec}$. Assume that the gravitational constant, $g = 10 \text{ m}/\text{sec}^2$.
- (a) Set up an initial value problem that describes this situation.
 - (b) Solve the initial value problem.
 - (c) What is the quasi-frequency of the system?

6. (14 points) Rewrite the following function in terms of step functions and find its Laplace transform:

$$f(t) = \begin{cases} t^2, & 0 \leq t < 2 \\ e^{3t} - t, & t \geq 2 \end{cases}$$

7. (12 points) Solve the initial value problem:

$$y'' + 2y' + 5y = 3\delta(t - 2\pi), \quad y(0) = 1 \quad y'(0) = 0.$$

8. (14 points)

(a) Find the general solution of

$$X' = \begin{bmatrix} 2 & 5 \\ 2 & -1 \end{bmatrix} X.$$

(b) Suppose $X(0) = \begin{bmatrix} -5 \\ \alpha \end{bmatrix}$ and $\lim_{t \rightarrow \infty} X(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, what is the value of α ?

9. (14 points) Solve the initial value problem:

$$X' = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix} X, \quad X(0) = \begin{bmatrix} -2 \\ 5 \end{bmatrix}.$$