NAME : ____________________

MATH 251
Summer 2002
Exam 2
July 17, 2002

ID : ____________________

INSTRUCTOR : ____________________

There are **9** questions on **8** pages. Please read each problem carefully before starting to solve it. For each multiple choice problem 4 answers are given, only one of which is correct. Mark only one choice. For partial credit questions, all work must be shown - **credit will not be given for an answer unsupported by work.**

NO CALCULATORS ARE ALLOWED.
PLEASE DO NOT WRITE IN THE BOX BELOW.
A TABLE OF LAPLACE TRANSFORMS IS ATTACHED.

1:__________
2:__________
3:__________
4:__________
5:__________
6:__________
7:__________
8:__________
9:__________
Total:__________

1. (6 points) Which of the following systems corresponds to the 2nd order differential equation:

$$2y'' + 4y' + 8y = 0$$

   (a) $\begin{cases} x_1' = x_2 \\ x_2' = -4x_1 - 2x_2 \end{cases}$

   (b) $\begin{cases} x_1' = 4x_1 + 2x_2 \\ x_2' = 2x_1 + 4x_2 \end{cases}$

   (c) $\begin{cases} x_1' = x_2 \\ x_2' = 4x_1 + 2x_2 \end{cases}$

   (d) $\begin{cases} x_1' = -x_2 \\ x_2' = -4x_1 - 8x_2 \end{cases}$

2. (6 points) Find the inverse Laplace transform for $s > a$ of:

$$\frac{-2s + 1}{s^2 - 2s + 5}$$

   (a) $-2e^{-t}\cos(2t)$

   (b) $-2e^{-t}\cos(2t) + 3e^{-t}\sin(2t)$

   (c) $-2e^{t}\cos(2t) + e^{t}\sin(2t)$

   (d) $-2e^{t}\cos(2t) - \frac{1}{2}e^{t}\sin(2t)$

3. (6 points) Find the Laplace transform of $u_{\pi/2}(t)\cos(2t)$. The following identity may be useful:

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

   (a) $-\frac{s}{s^2+4}e^{-s\pi/2}$
   (b) $-\frac{2}{s^2+4}e^{-s\pi/2}$
   (c) $\frac{1}{(s^2+4)}e^{-s\pi/2}$
   (d) $\frac{s}{s^2+4s+4}e^{-s\pi/2}$

4. (6 points) Suppose $f(t) = 2 - u_2(t) + t\big(u_3(t) - u_6(t)\big)$, what is $f(5)$?
   (a) 2
   (b) 4
   (c) 6
   (d) 7

5. (12 points) When using the method of undetermined coefficients to solve the following equation, what is the form of the particular solution? **Do not solve for the constants**.

$$y'' - y' - 2y = t^2 e^{3t} + 3e^{-t} + te^{2t}\cos(2t)$$

6. (18 points) A mass of $1kg$ stretches a spring $\frac{5}{8}m$. The system has a damping constant of $4\frac{kg}{s}$. The mass is pulled down $1m$ from its equilibrium position and released. You may take $g = 10\frac{m}{s^2}$.
   (a) Set up an initial value problem modeling this system.
   (b) Solve this initial value problem.
   (c) At what quasi-frequency does the system oscillate?
   (d) In the absence of damping (i.e. $\gamma = 0$), what is the systems natural frequency?

7. (12 points) Rewrite the following function in terms of unit step functions and find it's Laplace transform.

$$f(t) = \begin{cases} t+1 & 0 \leqslant t < 3 \\ t^2 + e^t & 3 \leqslant t \end{cases}$$

8. (20 points) Solve the initial value problem:

$$y'' - 4y' + 4y = 2\delta(t-4) \qquad y(0) = 1 \qquad y'(0) = 0$$

9. (14 points) Solve the initial value problem:

$$\overline{X}' = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \qquad \overline{X}(0) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$