This exam has 11 questions for a total of 100 points. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is in parentheses to the right of the question number.

**You may not use a calculator on this exam. Please turn off and put away your cell phone.**
1. (5 points) The inverse Laplace transform of \( F(s) = \frac{3s - 1}{s^2 - 2s + 10} \) is

(a) \( 3e^t \cos 3t + \frac{2}{3}e^t \sin 3t \),

(b) \( 3e^t \cos 3t - \frac{1}{3}e^t \sin 3t \),

(c) \( 3e^{-t} \cos 3t - e^{-t} \sin 3t \),

(d) \( 3e^{-t} \cos 3t - \frac{4}{3}e^{-t} \sin 3t \).

2. (5 points) Suppose \( f(t) = 2 - 9u_1(t) + u_4(t)t^2 - u_9(t)(10 - t^2) \). What is \( f(7) \)?

(a) 9

(b) 42

(c) 80

(d) 81
3. (5 points) Suppose \( y(t) \) is the solution of the first order linear initial value problem
\[
y' + 4y = u_{2\pi}(t) \sin(t - 2\pi), \quad y(0) = -2.
\]
What is \( Y(s) \), the Laplace transform of \( y(t) \)?

(a) \[ Y(s) = \frac{e^{-2\pi s}}{(s^2 + 1)(s + 4)} - \frac{2}{s(s + 4)}; \]
(b) \[ Y(s) = \frac{e^{-2\pi s}}{(s^2 + 1)(s^2 + 4)} + \frac{2}{s^2 + 4}; \]
(c) \[ Y(s) = \frac{se^{-2\pi s}}{(s^2 + 1)(s + 4)} + \frac{2}{s + 4}; \]
(d) \[ Y(s) = \frac{e^{-2\pi s}}{(s^2 + 1)(s + 4)} - \frac{2}{s + 4}. \]

4. (5 points) Which system of first order linear equations below is equivalent to the second order linear equation
\[
y'' + 5y' - 6y = 0?
\]

(a) \[ \begin{align*}
x'_1 &= x_2 \\
x'_2 &= -5x_1 + 6x_2
\end{align*} \]
(b) \[ \begin{align*}
x'_1 &= x_2 \\
x'_2 &= 6x_1 - 5x_2
\end{align*} \]
(c) \[ \begin{align*}
x'_1 &= -x_1 \\
x'_2 &= 6x_1 + 5x_2
\end{align*} \]
(d) \[ \begin{align*}
x'_1 &= 5x_1 - 6x_2 \\
x'_2 &= x_2
\end{align*} \]
5. (5 points) All of the following systems of linear equations have a critical point at \((0,0)\). Only one of the systems has a critical point that is (neutrally) **stable**, but not asymptotically stable. Which system is it?

(a) \[ \mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \mathbf{x} \]

(b) \[ \mathbf{x}' = \begin{bmatrix} 0 & -4 \\ -1 & 0 \end{bmatrix} \mathbf{x} \]

(c) \[ \mathbf{x}' = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \mathbf{x} \]

(d) \[ \mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{x} \]

6. (5 points) Suppose every solution of the linear system \( \mathbf{x}' = A\mathbf{x} \) satisfies the property \( \lim_{t \to \infty} \mathbf{x}(t) = \mathbf{0} \). Then the critical point \((0,0)\) must be

(a) asymptotically stable.

(b) (neutrally) stable, but not asymptotically stable.

(c) unstable.

(d) semistable.
7. (16 points) Rewrite the following piecewise continuous function $f(t)$ in terms of the unit-step functions. Then find its Laplace transform.

$$f(t) = \begin{cases} 
\pi - t^2, & 0 \leq t < 5 \\
4e^{-2t}, & 5 \leq t 
\end{cases}$$
8. (16 points) Use the method of Laplace transforms to solve the initial value problem

\[ y'' - 3y' - 4y = u_2(t) - u_6(t), \quad y(0) = 0, \quad y'(0) = 0. \]
9. (14 points)

(a) (12 points) Solve the initial value problem

\[ x' = \begin{bmatrix} 2 & 4 \\ -6 & -8 \end{bmatrix} x, \quad x(0) = \begin{bmatrix} 0 \\ 4 \end{bmatrix}. \]

(b) (2 points) Classify the type and stability of the critical point at \((0, 0)\).
10. (10 points) Consider the system of linear equations \( x' = Ax \). Suppose that \( A \) has eigenvalues \( r = 3 \pm 2i \), and one of the eigenvectors corresponding to the eigenvalue \( r = 3 + 2i \) is \( \begin{bmatrix} -2 \\ 1 - i \end{bmatrix} \).

(a) (8 points) Find a real-valued solution of this system that satisfies the initial condition \( x(0) = \begin{bmatrix} -2 \\ 2 \end{bmatrix} \).

(b) (2 points) What are the type and stability of the critical point at \((0,0)\)?
11. (14 points)
   
   (a) (6 points) Find the critical points (there are 3) of the following nonlinear system.
   \[
   \begin{aligned}
   x' &= x(x + y) \\
   y' &= y(2 - x + y)
   \end{aligned}
   \]
   
   (b) (8 points) Linearize the following nonlinear system about its critical point \((0, 2)\) and classify its type and stability.
   \[
   \begin{aligned}
   x' &= xy - 6x \\
   y' &= xy - 2x + y - 2
   \end{aligned}
   \]