MATH 251

Examination II

April 7, 2008

Name:________________

Student Number:________________

Section:________________

This exam has 11 questions for a total of 100 points. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is in parentheses to the right of the question number.

YOU MAY NOT USE A CALCULATOR ON THIS EXAM. PLEASE TURN OFF AND PUT AWAY YOUR CELL PHONE.

Do not write in this box.

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1. (5 points) The inverse Laplace transform of $F(s) = \dfrac{3s-1}{s^2-2s+10}$ is

   (a) $3e^t \cos 3t + \dfrac{2}{3}e^t \sin 3t$,

   (b) $3e^t \cos 3t - \dfrac{1}{3}e^t \sin 3t$,

   (c) $3e^{-t} \cos 3t - e^{-t} \sin 3t$,

   (d) $3e^{-t} \cos 3t - \dfrac{4}{3}e^{-t} \sin 3t$.

2. (5 points) Suppose $f(t) = 2 - 9u_1(t) + u_4(t)t^2 - u_9(t)(10 - t^2)$. What is $f(7)$?

   (a) 9

   (b) 42

   (c) 80

   (d) 81

3. (5 points) Suppose $y(t)$ is the solution of the first order linear initial value problem

$$y' + 4y = u_{2\pi}(t)\sin(t - 2\pi), \qquad y(0) = -2.$$

What is $Y(s)$, the Laplace transform of $y(t)$?

(a) $Y(s) = \dfrac{e^{-2\pi s}}{(s^2+1)(s+4)} - \dfrac{2}{s(s+4)}$;

(b) $Y(s) = \dfrac{e^{-2\pi s}}{(s^2+1)(s^2+4)} + \dfrac{2}{s^2+4}$;

(c) $Y(s) = \dfrac{se^{-2\pi s}}{(s^2+1)(s+4)} + \dfrac{2}{s+4}$;

(d) $Y(s) = \dfrac{e^{-2\pi s}}{(s^2+1)(s+4)} - \dfrac{2}{s+4}$.

4. (5 points) Which system of first order linear equations below is equivalent to the second order linear equation

$$y'' + 5y' - 6y = 0?$$

(a) $\begin{cases} x_1' = x_2 \\ x_2' = -5x_1 + 6x_2 \end{cases}$

(b) $\begin{cases} x_1' = x_2 \\ x_2' = 6x_1 - 5x_2 \end{cases}$

(c) $\begin{cases} x_1' = -x_1 \\ x_2' = 6x_1 + 5x_2 \end{cases}$

(d) $\begin{cases} x_1' = 5x_1 - 6x_2 \\ x_2' = x_2 \end{cases}$

5. (5 points) All of the following systems of linear equations have a critical point at $(0, 0)$. Only one of the systems has a critical point that is (neutrally) **stable**, but not asymptotically stable. Which system is it?

   (a) $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \mathbf{x}$

   (b) $\mathbf{x}' = \begin{bmatrix} 0 & -4 \\ 1 & 0 \end{bmatrix} \mathbf{x}$

   (c) $\mathbf{x}' = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \mathbf{x}$

   (d) $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{x}$

6. (5 points) Suppose every solution of the linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ satisfies the property $\lim_{t\to\infty} \mathbf{x}(t) = \mathbf{0}$. Then the critical point $(0, 0)$ must be

   (a) asymptotically stable.

   (b) (neutrally) stable, but not asymptotically stable.

   (c) unstable.

   (d) semistable.

7. (16 points) Rewrite the following piecewise continuous function $f(t)$ in terms of the unit-step functions. Then find its Laplace transfrom.

$$f(t) = \begin{cases} \pi - t^2, & 0 \leq t < 5 \\ 4e^{-2t}, & 5 \leq t \end{cases} .$$

8. (16 points) Use the method of Laplace transforms to solve the initial value problem

$$y'' - 3y' - 4y = u_2(t) - u_6(t), \qquad y(0) = 0, \qquad y'(0) = 0.$$

9. (14 points)

(a) (12 points) Solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 2 & 4 \\ -6 & -8 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 0 \\ 4 \end{bmatrix}.$$

(b) (2 points) Classify the type and stability of the critical point at $(0, 0)$.

10. (10 points) Consider the system of linear equations $\mathbf{x}' = \mathbf{A}\mathbf{x}$. Suppose that $\mathbf{A}$ has eigenvalues $r = 3 \pm 2i$, and one of the eigenvectors corresponding to the eigenvalue $r = 3 + 2i$ is $\begin{bmatrix} -2 \\ 1 - i \end{bmatrix}$.

(a) (8 points) Find a **real-valued** solution of this system that satisfies the initial condition $\mathbf{x}(0) = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$.

(b) (2 points) What are the type and stability of the critical point at $(0, 0)$?

11. (14 points)

(a) (6 points) Find the critical points (there are 3) of the following nonlinear system.

$$\begin{cases} x' &= x(x+y) \\ y' &= y(2-x+y) \end{cases}$$

(b) (8 points) Linearize the following nonlinear system about its critical point $(0,2)$ and classify its type and stability.

$$\begin{cases} x' &= xy-6x \\ y' &= xy-2x+y-2 \end{cases}$$