Name:_____

MATH 251 Student Number:_____

Midterm Exam II Instructor:_____

March 29, 2007 Section:_____

This exam has **9** questions for a total of 100 points. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.**

**THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.** At the end of the examination, the booklet will be collected.

**Do not write in this box.**

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1. (10 points) Use the Laplace transform to solve the equation $y'+2y = 2e^{4t}$, with initial condition $y(0) = -1$. **You must use the Laplace transform to receive any credit.**

2. (10 points) Solve the initial value problem

$$y'' + 4y = \delta(t-1), \quad y(0) = 1, \quad y'(0) = 0.$$

3. (12 points) Express the following piecewise continuous function in terms of step functions and then find its Laplace transform:

$$f(t) = \begin{cases} 2t & 0 \leq t < 2 \\ t^2 - t & 2 \leq t < 2 \\ 0 & t \geq 3. \end{cases}$$

4. (15 points) Find the Laplace transform of the following functions.

a)

$$e^{-3t}\cos 5t$$

b)

$$te^{5t}$$

c)

$$u_{\pi/2}(t)\sin t$$

Hint: Recall that $\sin(t+\pi/2) = \cos(t)$.

5. (10 points) Find the inverse Laplace transform of

a)

$$\frac{3s+4}{s^2+4s+20}$$

b)

$$\frac{e^{-8s}}{(s-9)^2}.$$

6. (5 points) Suppose the phase portrait of the system

$$x' = \begin{bmatrix} 2 & 1 \\ -1 & d \end{bmatrix} x$$

has an improper node at $(0, 0)$. Find the value (or range of values) of $d$.

7. (5 points) Find the general solution of the system

$$x' = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} x.$$

8. (20 points) Consider the system of linear equations

$$x' = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} x; \qquad x(0) = \begin{bmatrix} 1 \\ -4 \end{bmatrix}.$$

Solve the initial value problem and classify the type and stability of the critical point at $(0, 0)$.

9. (13 points) The displacement $y(t)$ of a mass-spring system is described by the equation

$$y'' + 4y' + 3y = g(t),$$

where

$$g(t) = \begin{cases} 1, & 0 \le t < 10 \\ 0, & t \ge 10. \end{cases}$$

At $t = 0$, the system is set in motion from equilibrium position with no initial velocity.

a) Find the displacement function $y(t)$.

b) What is the displacement when $t = 5$?