

MATH 251
Spring 2003
Exam 1
April 4, 2003

NAME : _____

ID : _____

INSTRUCTOR : _____

There are **10** questions on **9** pages. Please read each problem carefully before starting to solve it. For each multiple choice problem 4 answers are given, only one of which is correct. Mark only one choice. For partial credit questions, all work must be shown - **credit will not be given for an answer unsupported by work.**

NO CALCULATORS ARE ALLOWED.
PLEASE DO NOT WRITE IN THE BOX BELOW.

1: _____
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8: _____
9: _____
10: _____
Total: _____

1. (6 points) The second order linear equation $y'' - 2y' + 3y = 0$ is equivalent to

(a)
$$\begin{cases} x_1' = x_2 \\ x_2' = -3x_1 + 2x_2 \end{cases}$$

(b)
$$\begin{cases} x_1' = -x_1 \\ x_2' = 3x_1 - 2x_2 \end{cases}$$

(c)
$$\begin{cases} x_1' = x_2 \\ x_2' = 3x_1 - 2x_2 \end{cases}$$

(d)
$$\begin{cases} x_1' = x_1 \\ x_2' = -3x_1 + 2x_2 \end{cases}$$

2. (6 points) Which of the following is the Laplace transform of $t \sin(2t)$?

(a) $\frac{-2}{(s^2+4)^2}$

(b) $\frac{4s}{(s^2+4)^2}$

(c) $\frac{s}{s^2+4}$

(d) $\frac{3}{s^2(s^2+4)}$

3. (6 points) Let $g(t) = tu_1(t) - t^2u_3(t) + tu_5(t)$. What is $g(4)$?

(a) 4

(b) -12

(c) -16

(d) $\frac{e^{-s}}{s^2} - \frac{2e^{-3s}}{s^3} + \frac{e^{-5s}}{s^2}$

4. (6 points) What is the inverse Laplace transform of

$$\frac{2s + 7}{s^2 + 8s + 65}$$

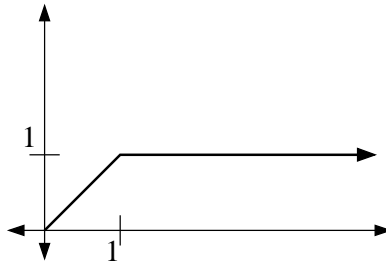
(a) $2e^{-8t} \cos t - 11e^{-8t} \sin t$

(b) $2e^{-4t} \cos 7t - \frac{1}{7}e^{-4t} \sin 7t$

(c) $2e^{-7t} \cos 4t - \frac{1}{4}e^{-7t} \sin 4t$

(d) $3 \cos 7t - 4 \sin 7t$

5. (6 points) Which of the following functions corresponds to this graph?



- (a) $tu_1(t) - 1$
- (b) $t + u_1(t)$
- (c) $(1 - t)u_2(t) + t$
- (d) $t + (1 - t)u_1(t)$

6. (14 points) A mass of $1kg$ stretches a spring $1m$. The system has a damping constant of $6\frac{kg}{s}$. The mass starts at it's equilibrium position with velocity $6\frac{m}{s}$. You may take $g = 10\frac{m}{s^2}$.

(a) Model this system.

(b) What is the quasi-frequency of this system?

7. (18 points) Rewrite the following function in terms of step functions and find its Laplace transform.

$$f(t) = \begin{cases} e^{2t} & 0 \leq t < 4 \\ t^2 - 1 & 4 \leq t \end{cases}$$

8. (10 points) Solve the following using Laplace transforms.

$$y'' + 2y' + 2y = \delta(t - 3) \quad y(0) = 0 \quad y'(0) = 0$$

9. (16 points)

(a) Find the general solution of

$$X' = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix} X$$

(b) If

$$X(0) = \begin{bmatrix} 2 \\ \alpha \end{bmatrix} \quad \text{and} \quad \lim_{t \rightarrow \infty} X(t) = [0 \ 0]$$

find α .

10. (12 points) Solve the initial value problem

$$X' = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix} X \quad X(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$