

MATH 251  
2nd exam  
Mar 27, 2000

Name: \_\_\_\_\_  
Student Number: \_\_\_\_\_  
Instructor: \_\_\_\_\_  
Section: \_\_\_\_\_

There are **7** partial credit questions. **In order to obtain full credit for these problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is in parentheses to the right of the question number.

**THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.** At the end of the examination, the booklet will be collected.

1: _____
2: _____
3: _____
4: _____
5: _____
6: _____
7: _____
Total: _____

**Do not write  
in the box  
to the left**

1. (15 points) Find the function  $y(t)$  whose Laplace transform is the expression  $\frac{7s^2 - 8s + 12}{(s - 2)(s^2 + 4)}$ .

2. (15 points) Rewrite the following function  $f(t)$  in terms of step functions, and find its Laplace transform (it is not necessary to graph the function):

$$f(t) = \begin{cases} t & : t < 2 \\ (t-2)^2 & : t \geq 2 \end{cases}$$

3. (15 points) A 4 kg mass is hung on a spring with spring constant 1 N/m. At time  $t = 0$  the mass is released from its equilibrium position, moving with an upward velocity of 1 m/s. Then at time  $t = \frac{\pi}{2}$ , an electromagnet is turned on, which exerts an upward external force  $\sin t$  on the mass.

Set up and solve the initial value problem for  $u(t)$ , the displacement of the mass from equilibrium at time  $t$ .

4. (15 points) Solve the initial value problem  $y'' + 4y = -2\delta(t - \frac{\pi}{4})$ ,  $y(0) = 2$ ,  $y'(0) = 0$ .

5. (a) (10 points) Find the eigenvalues of

$$\begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$$

- (b) Find the general solution of  $\vec{x}' = A\vec{x}$ .  
(c) Describe the type and stability of the critical point  $\vec{x} = \vec{0}$ .

6. (a) (10 points) Find the eigenvalues of

$$\begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$$

- (b) Find the general solution of  $\vec{x}' = A\vec{x}$ .  
(c) Describe the type and stability of the critical point  $\vec{x} = \vec{0}$ .

7. (20 points) Find the solution of the initial value problem

$$\vec{x}' = \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix} \vec{x} \quad \vec{x}(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$